Elastodynamic Shape Modeling in Virtual Medicine

Arne Radetzky[a], Andreas Nürnberger[b], Michael Teistler[a], Dietrich Peter Pretschner[a]

[a]Institute for Medical Informatics, Technical University of Braunschweig,
Fallersleber-Tor-Wall 22, D-38100 Braunschweig, Germany
A.Radetzky@umi.cs.tu-bs.de, Tel. +49 531 391 2125, Fax +49 531 391 9502

[b]Institute of Knowledge Processing and Language Engineering, University of Magdeburg,
Universitätsplatz 2, D-39106 Magdeburg, Germany
andreas.nuernberger@cs.uni-magdeburg.de, Tel. +49 391 67 11358, Fax +49 391 67 12018

Abstract

Surgical simulation is the coming training method for medical education. The main reasons for this are the reduced risk for the patients and the easy repeatability of complicated surgical procedures. Therefore, an improved impression of reality during the simulated training must be obtained. For this, a complex model of the human's anatomy and physiology is needed. With regards to pathological conditions, which should be considered, it is necessary to build more general anatomical models. Simple static models are unsuitable for surgical simulation because convincing interactivity is only possible with deformable organs and elastic tissues. Traditional models of tissue deformation have difficulties to simulate the appearance of deformation because of the unknown physical parameters of the tissue's elasticity. Hence this paper describes a method for elastodynamic shape modeling with neuro-fuzzy systems, which are able to adapt the necessary parameters from real tissues.

1. Introduction

During training for aircraft pilots it is taken for granted to complete many successful flights in a simulator before flying a real airliner. This is reasonable because of the increasing complexity of airliners and the intensifying need of the passenger's security. However, in the hazardous area of surgery the medical training is performed directly on the patient after using simple models built of animal organs or corpses. With surgical simulators for medical training the quality of surgical interventions could be increased while simultaneously reducing the risk for the patient.

Nowadays computer assisted surgery is already used for procedure training and operation planning. Most of the utilized methods are based on static visualization techniques. To improve the benefit for surgical training a visual convincing static modeling of the operation scenario and the involved tissues is not sufficient, because tissues can be deformed at contact and transections can be made. Especially for the interaction with medical devices, such as laparoscopic instruments, it is necessary to simulate the deformation of tissue under the influence of collision forces.

The elasticity of structures can be described by use of differential equations and physical laws [5]. In many cases, a model of a physical system can not be obtained by conventional approaches. This can be caused by problems in constructing a system of differential equations and in finding the required parameters. Furthermore, the simulation of such a model description is often very time-consuming and so, the use in real time applications is sometimes impossible. A further problem is that often even small or local changes to the model structure require a rebuild of the whole shape model. Some approaches try to resolve these problems by pre-processing elementary deformations (e.g. [2]), but this is very expensive for complex objects.

Different approaches use artificial neural networks to simulate the behavior of physics-based model structures (e.g. [6]). These approaches have the disadvantage that the simulated structures could not be cut in smaller objects during the simulation process, which is necessary for the simulation of surgical training procedures. Besides the neural networks have to be trained off-line by use of observation data of physics based models, even if some expert knowledge about the physical model is available.

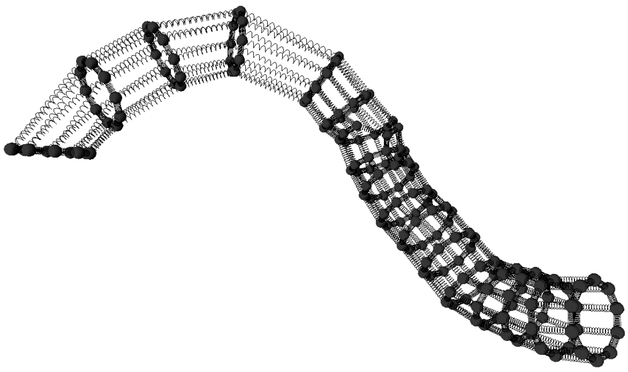
For these reasons, we developed a model, which is motivated by a combination of a fuzzy system and an artificial neural network, a so-called hybrid neuro-fuzzy system [12][19]. By using a fuzzy system, it is possible to integrate expert knowledge in form of medical terms to define the visual response of the tissue's shape model. In addition, with artificial neural network techniques it is possible to learn or adapt the parameters of the developed model. Measured data or physical models of the tissue can be used for learning.

In the following, we present parts of our actual work on this project. The underlying structure of the model is motivated and its implementation is presented. Furthermore, learning approaches are shown, and an example of a gynaecological laparoscopy in a virtual simulation environment is given.

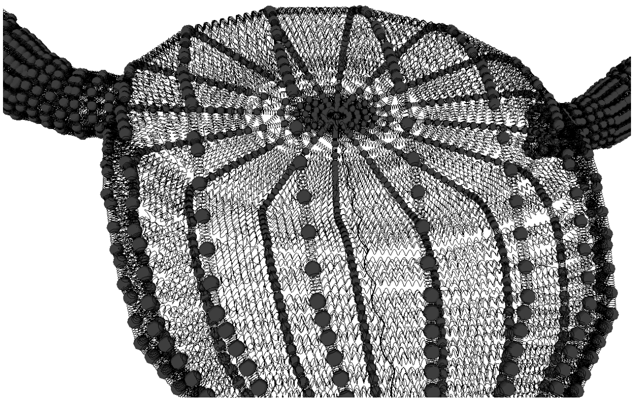
2. Dynamic shape modeling with spring-mass models

Most human organs can be described as enclosed shapes. These shapes can be created as static models using appropriate modeling software. To simulate deformable organs a dynamic component is needed. Every tissue has an assigned mass, elasticity and viscosity. To describe these physical conditions a spring-mass model can be used.

In spring-mass models the whole mass of the organs is divided up between the mass points of the model, which are represented as nodes in a mesh. Every node of the mesh can be connected elastically or viscously to its neighbors (see Figure 1). The connections are represented as springs. One advantage of this approach is, that transections can be simulated by disconnecting springs. The springs along the transection line are split up and two new nodes are connected to each of the loose endings of the springs (see Figure 2). Another advantage is, that the visualization of the shape can be done by using existing graphic algorithms e.g. texture mapping and surface rendering.

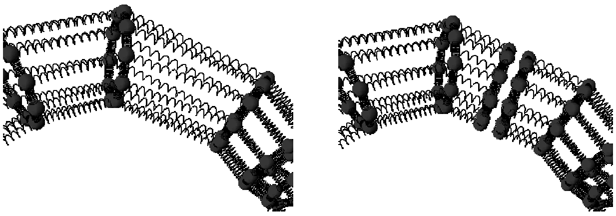


(a) Fallopian tube (tuba uterina)



(b) Uterus

Figure 1: Elastodynamic shape models of human anatomy: Mass points are connected via springs



(a) *(b)*

Figure 2: Transection of the fallopian tube

To every node of the mesh, external forces can be applied (for example gravity or collision forces). The internal forces and the displacement of the nodes can be computed correspondingly. Also, the displacement of a node can be given and the resulting forces can be calculated. By use of this model structure inertia and torque (e.g. in case of object rotation) can be simulated as well [17].

The use of surface meshes to simulate the organ's shape is sufficient in most cases (e.g. vessels). Same as in real anatomy, the outer shapes of the fallopian tube and the uterus have to be fixed because of the connection with the pelvic cavity. Hence, the mass nodes of the model are connected with it via springs. For other organs (e.g. a gallbladder) the shape can be stabilized with additional springs at the inside.

One of the first applications of spring-mass models for the simulation of elastic deformable models was developed by Terzopoulos et. al. [23]. Some improved techniques were presented, for example, in [2][3][22][21]. One of the main problems of these approaches is the difficulty to derive the parameters of the physical model and the high computational demands during simulation. To resolve these problems, a neural network architecture was developed, which is able to simulate spring-mass models [13]. In this way it is possible to learn the parameters of the physical model and to speed up the simulation by use of problem specific propagation procedures. Furthermore, a fuzzy system was implemented to initialize the network parameters if some prior knowledge about the model, like stiffness, elasticity or shiftability, is available [18]. The structure of this neuro-fuzzy approach is described in the following section.

3. Simulation of elastodynamic shapes with neuro-fuzzy systems

Fuzzy systems and neural networks are successfully used in the area of control theory, data analysis, and knowledge based systems [9][11][12]. Fuzzy systems can be used to derive parameters of dynamic systems, if only vague data about the system is available. Artificial recurrent neural networks can be used to simulate the dynamic of time-dependent systems. Furthermore, neural networks can be trained to simulate the behavior of real dynamic systems.

Neuro-fuzzy systems combine the advantages of both techniques, particularly the ability to learn of neural networks with the interpretability of fuzzy systems [12][19]. Therefore, a hybrid approach for the description and simulation of elastic tissue in virtual medicine was chosen.

The network parameters can be derived with the fuzzy-system if some prior knowledge is available, which can be used to define the behavior of the simulated tissue. The simulation process can be performed with the artificial recurrent neural network. Furthermore, it can be used to learn or adapt the parameters of the network, if measured data exist.

Since both systems can be used independently, the neural network and the fuzzy system are described separately in the following sections.

3.1. The network model

The presented network uses a problem specific structure. The structure of the neural network was designed to speed up the simulation and learning process for elastic solids. In contrast to common neural network models, this model uses vectors instead of single input and output signals. So the standard model of a neuron was vectorized.

The structure implements the system of differential equations (see, for example, [5] or [25]), which defines the spring-mass model. The presented model is capable to simulate nonlinear dynamics by use of arbitrary activation functions. Thus, it resolves the insufficient biomechanical realism of linear spring-mass models. For a comparison of different elasticity models see, for example, [2].

Two different network nodes are used for this purpose: the mass nodes and the spring nodes. In the following a short description of the model structure is given. For a more formal definition see [13].

Mass point dynamic. The neurons determining the mass dynamics (inertia) are divided in three 'sub-neurons' which calculate the position, velocity, and acceleration of the mass point (see Figure 3).

An external force can be applied to the neuron, which calculates the actual acceleration. The velocity and position neurons are self-connected feedback nodes. These neurons are used as integrators.

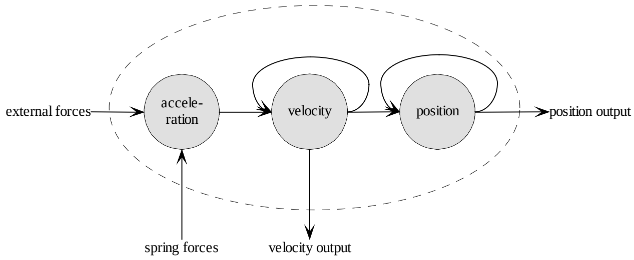


Figure 3: Neurons describing the mass point dynamic

Spring dynamic. The neurons determining the spring dynamics (see Figure 4) calculate the actual total spring force *F*, based on the position and velocity of the connected nodes.

The spring and viscosity functions for the calculation of the force *F* are implemented by the respective neurons. This force is defined as $F = f(p,v) + d(p,v)$, where $f(p,v)$ defines the spring force and $d(p,v)$ the damping force or viscosity.

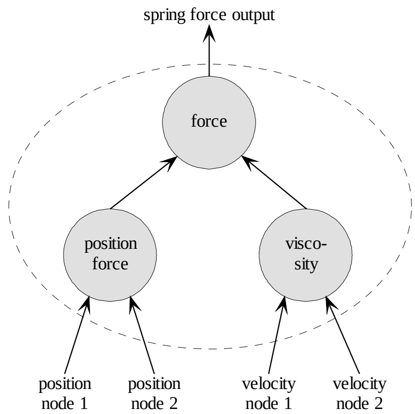


Figure 4: Neurons describing the spring dynamic

The network structure. The network is structured by alternating spring and node layers. A sample of a 2D-mesh is shown in Figure 5. Of course, different network structures can be used. Therefore, the network can be adapted for specific objects or to minimize the number of springs and nodes required for an appropriate simulation (see also section 2).

The system of differential equations, which is defined by the network structure, is solved during the propagation process. The used propagation methods are presented in the following section.

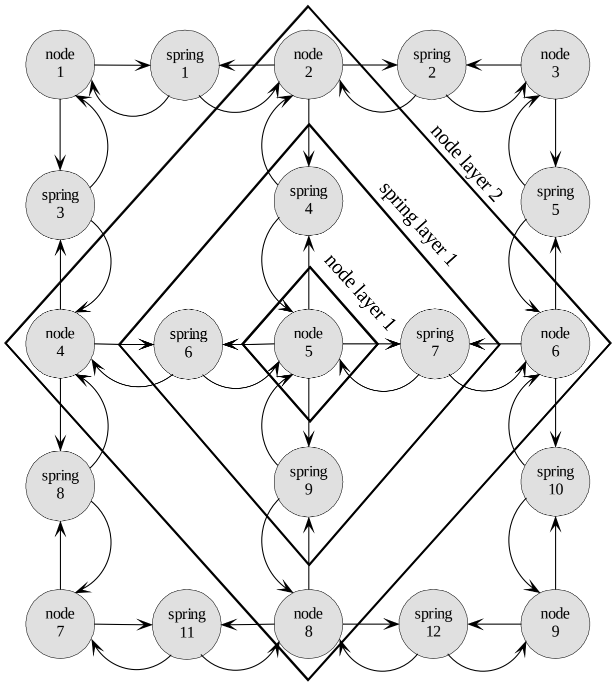


Figure 5: Network representation of a 2D-mesh

Propagation. In a first approach [17] we choose an algorithm similar to the propagation in Hopfield networks [8]. This propagation algorithm was separated in two steps. During the first step, the network was propagated until a local energy minimum was reached. The second step was used to calculate the activities of the sub-neurons for the next time step.

However, with rigid tissues stability problems could occur during the propagation process. If the parameters of the network are badly defined (e.g. large spring constants, very small viscosity) the network tends to an unstable (chaotic) behavior. For example, in case of great force values and large spring constants, the calculated displacement of the nodes during one time step could be much too far. Thus, the propagation process could start to oscillate or even diverge. Choosing a very low time constant can solve the problem. But then, the propagation process can not be done in real time.

However, experiments have shown that unstable behaviors only occur during very short time intervals, so that a variable time step can be used. This idea was implemented in our second approach [18].

The computation can be done parallel with all nodes and springs. To compute this network it is effective to group several nodes and springs to greater units and solve each unit on a single processor. The order of computation of the node and spring layers during propagation is important to minimize the required steps. Therefore, a heuristic is used to determine the order of computation. During the first propagation, the node with the greatest force vector is identified. Then the next propagation starts with this node and continuously goes on layer by layer until all nodes are computed. During the propagation, the node with the greatest force difference between the old force and the new computed force is identified. If the force difference is greater than a threshold value, the propagation starts over.

During propagation, the network is updated using the time constant t_Δ. In real time applications, this constant can be used to define the graphic refresh rate. Thus, it is possible to synchronize the propagation process with a real time environment.

The learning and initialization methods. We are currently working on learning methods, which are based on backpropagation learning methods for recurrent neural networks (see, for example, [7][11][14][15]).

The learning algorithms use measured data or data generated by an exact physical model of the tissue for learning. The position of every node at discrete time-steps is used as input. The learning algorithm tries to minimize the error (total cost) function of the network. The parameters (weights) of the network are adapted after each time-step by a gradient descent method. The learning process is finished if the error is sufficient small. Thus, even local differences in the solid structure can be learned. During learning only the weights of the velocity sub-neurons (the masses) and the spring neurons are modified to ensure the interpretability of the network [13].

If available, the parameters of the real physical model can be used to set the weights of the neural network directly or to initialize the learning process. Furthermore, the initialization can be done by use of a fuzzy system [9][12].

3.2. The fuzzy system

The fuzzy system can be used to describe the relations between existing (vague) expert knowledge of the solid behavior (for example ‘very hard’, ‘soft’, ‘elastic’) and the network parameters [17].

The fuzzy sets and rules can be edited with the program ‘Elastodynamic Shape Modeler’ [16] shown in Figure 7 (in color section). The ‘Fuzzy Rules Window’ shows some sample fuzzy rules. The fuzzy rules were derived by inquiring of experts and they are optimized manually. Currently, we are working on neuro-fuzzy methods to optimize the derived rule base (see, for example, [10][12]).

The fuzzy system defined this way makes it possible to derive the parameters of the network by use of linguistic terms or crisp input values, which can be selected, for example, by sliders (see Figure 7 in color section, ‘Fuzzy Input Window’). Of course, specific parts of the network can be defined separately to make the simulation of tissue consisting of different layers possible. This can be done by creating an arbitrary ellipsoid, which defines the region of parameter changing. Variations of the fuzzy system result in different behavior of the simulated object.

4. Creation of elastodynamic shapes

One main difficulty developing surgical simulators is to create appropriate anatomical models. Because of the wide differences between the appearance of varying pathological conditions, these models must be created with a 3D CAD- or modeling tool. However, the created models can only be used as static shapes. Nevertheless, to utilize these models in surgical simulation a conversion to elastodynamic shapes must be performed.

Nearly all 3D models, which are created by modeling software, consist of sets of connected triangles or triangular meshes. The edges of these triangles can be converted to springs and the vertices to nodes of the spring-mass model (e.g. the model of the uterus in Figure 1 *(b)*). Using traditional methods to simulate deformable objects, this direct conversion is not possible because the resulting appearance of deformation depends on the structure of the used spring-mass model to a great part. However, using neuro-fuzzy systems for simulation, the deformation behavior can be learned from real tissues regardless of the model's structure [13]. Of course, this is only possible if the organs are connected with other organs or cavities. For organs, which are not connected, it might be necessary to insert additional springs for the stabilization of the outer shape. These springs can be added as lines by use of a modeling tool.

To simplify the development process, we have developed the program 'Elastodynamic Shape Modeler' (see Figure 7 in the color section), which transforms triangular meshes and line sets into spring-mass models. The geometric description of the shapes can be imported directly from the used modeling tool. A fuzzy rule base is used to modify the parameters of the elastodynamic shapes in an arbitrary defined ellipsoid region. The deformation behavior is shown in a graphical window. In addition the occurring forces of the shape's deformation can be felt by use of a force feedback device.

5. Simulation of elastodynamic shapes

The spring-mass models of the elastodynamic shapes can be simulated as a whole. This is called *total deformation* and can be used especially if the objects are not fixed or connected with other objects. The tuba uterina shown in Figure 12 (in the color section) is simulated with total deformation. However, the total deformation can only be used in real time with small shapes consisting of only a few hundred nodes and springs. Especially shapes, which are created by a modeling software have up to a few hundred thousands triangles. Even if the number of triangles is reduced by use of simplifying algorithms for triangular meshes, the mesh is usually too complex to simulate the elastodynamic shape in real time. The uterus at Figure 1 *(b)* with about 5000 springs and nodes is an example of such a complex shape.

However, most anatomical structures in the human body are fixed and connected with other organs. Therefore, it is not necessary to simulate the whole elastodynamic shape.

For example, to simulate the deformation of the uterus, which consists of very tight tissue, it is sufficient to use only a global deformation. Near the barycenter of the object a simple spring-mass model consisting of three springs and four nodes is created (see Figure 6 *(a)*). The nodes are consecutively connected by springs. Node 1 and node 4 are fixed and can't move. The initial positions of node 1 and node 2 are the same as the position of node 3 and node 4. The position of the object depends on the center of the spring between the inner nodes 2 and 3. The orientation of the spring specifies the orientation of the object. In case of a collision between a surgical tool and the object a node and two springs are created. These springs connect the new node with the inner nodes. They are used to calculate the external forces of the inner nodes (see Figure 6 *(b)*).

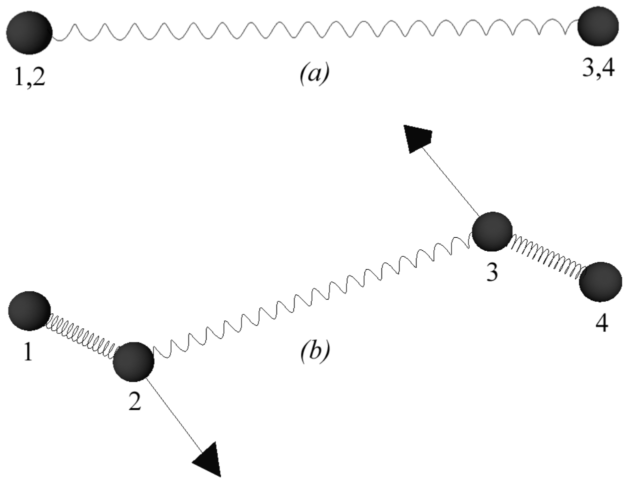


Figure 6: Simple elastodynamic model for global deformation (*(a)* undeformed, *(b)* deformed)

The global deformation is very fast, because only a few springs are needed for simulation. However, only tight tissues can be simulated that way. Additionally a *local deformation* can be combined with the global deformation. A local deformation is the same as a total deformation except that the local deformation is limited to the part of the organ, which has contact with the instrument. This can be done, for example, with a recursive propagation algorithm starting from the center of collision. For the use in the surgical simulator all deformation techniques are combined to get a maximum frame rate and a highly realistic impression.

6. Application example

Although the simulation of deformable tissues can be used in a wide field of surgical training (e.g. [1][2][3][4][20]), the main field of application is the laparoscopy also known as keyhole surgery. On the one hand, force feedback devices for the simulation of laparoscopic instruments already exists. On the other hand, keyhole surgery requires much more training as conventional surgery.

We choose the gynaecological laparoscopy as the main field of application for our simulation system SUSILAP-G (**SU**rgical **SI**mulator for **LAP**aroscopy in **G**ynaecology). Many surgical interventions in gynaecology aim at operations of the tuba uterina. Thus, for the first application example a sterilization was chosen, which can be performed with SUSILAP-G. An endoscopic view of a real sterilization can be seen in Figure 8 in the color section.

The simulator is written in C++ and the graphical interface is based on Open Inventor [24], a standard object orientated graphic library, which is available for many operating systems. The elastodynamic shapes and the simulation with neuro-fuzzy systems are fully embedded in the class library of Open Inventor. Currently we have developed versions for Windows and Unix.

An example of the virtual simulation environment of SUSILAP-G is shown in Figure 9 in the color section. A female patient in an operation theater is presented. In this environment, two virtual laparoscopic instruments can be manipulated using Laparoscopic Impulse Engines, which are installed in a dummy (see Figure 10 in the color section). The Impulse Engines can be moved like real laparoscopic instruments with four degrees of freedom. The virtual operation can be observed on a computer monitor. If virtual organs are deformed with the instruments, the resulting forces can be felt.

On the monitor, the view of the virtual endoscope can be seen. The enlarged endoscopic view is shown in Figure 11 in the color section. The uterus can be seen at the right side and the ileum (a part of the intestine) on the left side of the picture. The tuba uterina can be coagulated with coagulation forceps. Other instruments like scissors and needles can also be chosen. The fallopian tube is build by use of 400 nodes and 1200 springs and is simulated with total deformation. For manipulations of the fallopian tube it is sufficient to simulate the uterus with a local deformation, since its global position remains nearly unchanged during operation.

The ileum is simulated using global deformation and simple ellipsoid shapes. Adjacent shapes are connected to each other by just one spring. Every shape is simulated using global deformation. Thus, the total number of springs can be reduced without decreasing the impression of reality.

We are currently using a Silicon Graphics Onyx2 Infinite Reality for high-end visualization. Only one CPU is used for the elastodynamic simulation of the complete surgical intervention shown in Figure 12 in the color section.

7. Summary

In the last section we have shown that the developed model can be successfully used for the simulation of deformable shapes e.g. tissues in surgical simulation. The shapes can be created with standard modeling software and they can be converted to spring-mass models.

These models can be simulated by use of global deformation if the represented organ is nearly rigid or if it is not the main organ to be operated on. Otherwise, a fast simulation technique based on local deformation can be used. This technique is especially useful for the simulation of organs because most organs are connected with cavities, so simulating a total deformation is not necessary.

By use of the presented model, it is easy to apply external forces to any node, for example, gravity forces or collision forces caused by medical tools. Furthermore, changes of the object's structure can be done during simulation, for example changes caused by transections, ruptures or fractions. The weights of the network can be initialized by real mass and spring parameters. Besides, these parameters can be adapted or learned by use of a physical model or measured data of real objects. Besides, the parameters can be defined by means of simple rules, which are determined by inquiring medical experts. With the program 'Elastodynamic Shape Modeler' the experts can observe and feel the deformation resulting from the use of these rules.

Currently we are working on improvements for the learning procedure. One of the objectives of our project is to develop a complete method, which enables us to measure the elastodynamic behavior of real tissues and to generate a simulation model of this tissue, by use of the measured data. For the use in surgical simulation real time three-dimensional data of deformed organs are difficult to achieve. At present, we are working on non-invasive methods to obtain these data by the use of sonography.

Up-to-date information concerning this project can be obtained via the Internet from http://www.umi.cs.tu-bs.de/ara or http://fuzzy.cs.uni-magdeburg.de/~nuernb.

References

[1] Basdogan, C.; Loan, P.; Rosen, J.M. et al., An Interactive Model of the Human Thigh for Simulation Surgical Procedures in Virtual Environments. In *Proc. of the Winter Annual Meeting of ASME*, 1996.

[2] Cotin, S.; Delingette, H.; Ayache, N., Efficient Linear Elastic Models of Soft Tissues for real-time surgery simulation, *INRIA report no. 3510*, INRIA Sophia Antipolis, France, 1998.

[3] Cotin, S.; Delingette, H.; Ayache, N., Real-time elastic deformations of soft tissues for surgery simulation, *INRIA report no. 3511*, INRIA Sophia Antipolis, France, 1998.

[4] Delp, S. L.; Loan, P.; Basdogan, C. et al., Surgical Simulation: An Emerging Technology for Training in Emergency Medicine. In*: Teleoperators and Virtual Environments 6 (1997) 4*, pp. 147-159.

[5] Gould, P. L., *Introduction to Linear Elasticity*, Springer, New York, 1994.

[6] Grzeszczuk, R.; Terzopoulos, D.; Hinton, G., NeuroAnimator: Fast neural network emulation and control of physics-based models, *Proc. of ACM SIGGRAPH 98*, Orlando, FL, July, 1998, in Computer Graphics Proceedings, Annual Conference Series, 1998, pp. 9-20.

[7] Hayken, S., *Neural Networks*, Prentice-Hall Inc., New Jersey et al., 1994.

[8] Hopfield, J. J., Neural networks and physical systems with emergent collective computational abilities. In: *Proc. Nat. Acad. Sci. (Vol. 79)*, 1982, pp. 2554-2558.

[9] Kruse, R.; Gebhardt, J.; Klawonn, F., *Foundations of Fuzzy Systems*, John Wiley & Sons Inc., New York et al., 1994.

[10] Kruse, R.; Nürnberger, A., Learning Methods for Fuzzy Systems, In: *Non-Linear Electromagnetic Systems: Advanced Techniques and Mathematical Methods*, Edited by V. Kose and J. Sievert, IOS Press, Amsterdam, Netherlands, 1998.

[11] Lin, C. T.; Lee, C. S. G., *Neural Fuzzy Systems: A Neuro-Fuzzy Synergism to Intelligent Systems*, Prentice-Hall Inc., New Jersey et al., 1996.

[12] Nauck, D.; Klawonn, F.; Kruse, R., *Foundations of Neuro-Fuzzy Systems*, John Wiley & Sons Inc., New York et al., 1997.

[13] Nürnberger, A.; Radetzky, R.; Kruse, R., A Problem Specific Recurrent Neural Network for the Description and Simulation of Dynamic Spring Models, In: *Proc. of the International Joint Conference on Neural Networks*, 1998, pp. 468-473.

[14] Pineda F. J., Generalization of back-propagation to recurrent neural networks. In: *Physical Review Letters 59 (1987) 19*, pp. 2229-2232.

[15] Pineda F. J., Recurrent back-propagation and the dynamical approach to adaptive neural computation, In*: Neural Computation (1989) 1*, pp. 161-172.

[16] Radetzky, A.; Machenski, H.; Pretschner, D. P., A Fuzzy Parameter Estimator to Describe the Elastic Behavior of Deformable Tissues in Surgical Simulation. In: *Scientific Program of the RSNA (1998) 209(P)*, p. 681.

[17] Radetzky, A.; Nürnberger, A.; Pretschner, D. P., A Neuro-Fuzzy Approach for the Description and Simulation of Elastic Tissues. In: Baumeister, M.; Hohn, H. P.; Sklorz, S. et al. (eds.): *Multimedia Technology in Medical Training, Proc. of the European Workshop*, 1997, pp. 30-40.

[18] Radetzky, A.; Nürnberger, A.; Pretschner, D. P., Simulation of elastic tissues in virtual medicine using neuro-fuzzy systems. In: Kim, Y.; Mun, S. K. (eds.): *Medical Imaging 1998: Image Display. Proc. of SPIE Vol. 3335*, 1998, pp. 399-409.

[19] Rojas, Raúl, *Neural Networks - A Systematic Introduction*, Springer, Berlin et al.: 1996.

[20] Suzuki, N.; Hattori, A.; Kai, S. et al., Surgical Planning System for Soft Tissues Using Virtual Reality. In: Morgan, K.S. (edt.): *Medicine Meets Virtual Reality: Global Healthcare Grid, Vol. 39 of Studies in Health Technology and Informatics*, IOS Press, Amsterdam et al., 1997, pp. 159-163.

[21] Terzopoulos, D.; Fleischer, K., Deformable Models, *The Visual Computer (1988) Vol. 4*, pp. 306-331

[22] Terzopoulos, D.; Fleischer, K., Modeling inelastic deformation: Viscoelasticity, plasticity, fracture, In: *Computer Graphics, 22(4), Proc. ACM SIGGRAPH'88 Conference*, Atlanta, GA, August, 1988, pp. 269-278.

[23] Terzopoulos, D.; Platt, J.; Barr, A.; Fleischer, K., Elastically deformable models, In: *Computer Graphics, 21(4), 1987, 205-214, Proc. ACM SIGGRAPH'87 Conference*, Anaheim, CA, July, 1987. Translated to Japanese by Nikkei-McGraw-Hill and published in Nikkei Computer Graphics, 3(18), 1988, pp. 118-128.

[24] Wernecke, J., *The Inventor Mentor*. Addison-Wesley, New York et al., 1994.

[25] Wesolowski, Z. (edt.), *Nonlinear Dynamics of Elastic Bodies*, Springer, Wien, 1978.