

Exercise Sheet 1

Exercise 1 Combinatorics: Number of Possibilities

- a) 6 white, 4 black and 2 red socks shall be pegged out to a clothesline for drying. Socks with the same color are indistinguishable. How many different arrangements of the socks are possible?
- b) How many possibilities are there for distributing 9 indistinguishable balls over 4 boxes?
- c) How many possibilities exist for forming a committee consisting of 3 women and 2 men if there are 5 women and 5 men to choose from?

Exercise 2 Probabilities: Rolling Dice

Consider the following die game between the banker and the player: The banker rolls two icosahedra ("dice" with 20 faces) whereas the player rolls just one icosahedron. The player wins if his number lies properly between the banker's two numbers, that is, the bank wins if the player's number is equal to one banker's number or smaller or larger than both banker's numbers. What is the player's probability of winning the game?

Exercise 3 Probabilities: Rescuing an Astrologer

An astrologer once angered his King which resulted in a death sentence to be executed by the hangman. However, the King offered him a chance to pardon. The astronomer was given two white and two black balls which he was asked to distribute over two urns. The hangman would choose one urn arbitrarily and pick from it a ball at random. If this ball turns out to be white, the astronomer is pardoned. If it is black, he will be executed on the spot. What is the best strategy of distributing the balls for the astronomer to maximize his chances of survival? What is the probability of survival then? How should he act if he was given three white and three black balls in the beginning? What can be said about the general case of being given n black and n white balls?

Exercise 4 Geometric Probabilities

- a) A stick of one meter length is sawn through at two randomly picked points. What is the probability of being able to construct from the three parts a triangle? (Hint: Consider the lengths of the three parts as coordinates in three-dimensional space and exploit the triangle inequality in all its forms.)
- b) A secant is drawn into a circle at random. What is the probability of this secant being longer than the side of the equilateral triangle inscribed into the circle?

Additional Exercise Variants of the Monty Hall Problem

The original form of the Monty Hall Problem should be eventually known: At a game show, the candidate is presented three closed doors. Behind one of them there is the prize. The candidate chooses one door (which is not yet opened) whereafter the host opens one of the remaining two doors behind which there is no prize. Now, the candidate is offered the opportunity to reconsider, that is, he may change the doors. The door he finally chose is opened. How should he act?

Since this problem (and thus its solution) is widely known, we will consider a variant here. Instead of three doors we now deal with *seven* doors. In the beginning the candidate is allowed to choose *three* doors. From the remaining four doors, three without the prize behind are opened by the host. Again, the candidate is given the opportunity to drop his initially chosen doors and switch to the remaining door. How should he act to maximize his chance of winning?¹

Yet another variant, with two intermediate steps: The candidate has to choose one out of four doors. The host then opens one of the three remaining doors behind which there is no prize and offers the candidate to change his decision. After this decision the host again opens one of the remaining doors that are currently not chosen by the candidate and do not contain the prize. Then, again, he offers the candidate to reconsider. Which strategy maximizes the chance of winning?

¹If he rejects to switch, that is, if he sticks to his initially chosen three doors, all three are opened in the last step.