

## Exercise Sheet 2

### Exercise 5 Conditional Probabilities

- Four balls are placed into four boxes one after another. All  $4^4$  orders be equally likely. What is the probability that a box contains exactly three balls given the fact that the first two balls have been placed into different boxes?
- A family has two children. What is the probability of both being girls if is known that at least one of them is a girl?
- What is the probability of b) if it is known that the younger child is a girl?

### Exercise 6 Stochastic Independency

- A wheel of fortune has 36 numbered sectors (numbers 1 to 36). These sectors are colored in red (R) or blue (B) according the following scheme:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
R	R	R	R	R	B	B	B	B	R	R	R	R	B	B	B	B	B
36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19

We consider the three events

- $A$ : the wheel stops in a red sector,
- $B$ : the wheel stops in a sector with an even number,
- $C$ : the wheel stops in a sector with a number  $\leq 18$ .

Show that these events are pairwise independent but that  $P(A \cap B \cap C) = P(A)P(B)P(C)$  does not hold.

- Two fair dice, red and white, are cast. We consider the following three events:

- $A$ : the red die shows up 1 or 2,
- $B$ : the white die shows up 3, 4 or 5,
- $C$ : the sum of the spots of both dice is 4, 11 or 12.

Show that  $P(A \cap B \cap C) = P(A)P(B)P(C)$  holds but not the pairwise independence.

**Exercise 7**      Bayesian Theorem

- a) A weather satellite sends a binary encoded description of a developing storm. Inevitable noise (atmospheric interferences) causes transmission errors. Assume the message contains 70% zeros and the probability of correctly receiving a sent bit is 80%. What is the probability of having sent a zero if a one was received?
- b) Color blindness affects 5 out of 100 men and 25 out of 10000 women. A color blind person is randomly picked. What is the probability of this person being male?

**Exercise 8**      Bayesian Theorem

- a) In a given population, 2% of all persons suffer a certain disease. Let a test have the property that it correctly recognizes an ill person with 95% probability whereas the rate of correctly revealing a healthy person is 99%. What is the probability that a person does (not) suffer from the disease if the test does (not) reveal the disease?
- b) Consider two urns. Urn 1 contains two white and one red ball, urn 2 one white and two red. First, a ball from urn 1 is randomly chosen and placed into urn 2. Finally, a ball from urn 2 is picked. This ball be red: What is the probability that the ball transferred from urn 1 to urn 2 was white?

**Additional Exercise**      Probabilities: Triple Duel

*A*, *B* and *C* compete against each other in a duel with pistols simultaneously. *A* is the worst shooter: His chance of hitting the target is 0.3. The chance of *C* is 0.5 whereas *B* never misses his target. The three shot in order *A*, *B*, *C*, *A*, ... at a target of their choice (of course, who was shot, quits the game) until just one of them is left. Which is *A*'s best strategy?