

## Assignment Sheet 8

### Assignment 28      Fuzzy Relational Equations

Let  $X = \{x_1, x_2, x_3\}$  and  $Y = \{y_1, y_2, y_3, y_4\}$  be two sets and  $\mu : X \rightarrow [0, 1]$  and  $\nu : Y \rightarrow [0, 1]$  two fuzzy sets on  $X$  and  $Y$ , respectively, which are defined as follows:

$$\begin{aligned} \mu(x_1) &= 0.1, & \mu(x_2) &= 0.7, & \mu(x_3) &= 1.0, \\ \nu(y_1) &= 0.4, & \nu(y_2) &= 1.0, & \nu(y_3) &= 0.8, & \nu(y_4) &= 0.3. \end{aligned}$$

- a) How can you find out whether the relational equation  $\mu \circ \varrho = \nu$  has a solution, *i.e.* whether there is a fuzzy relation  $\varrho$  that satisfies this equation?
- b) If the relational equation  $\mu \circ \varrho = \nu$  has a solution, determine a solution. Are there other solutions than the one you found?

### Assignment 29      Fuzzy Relational Equations

Let  $X = \{x_1, x_2, x_3\}$  and  $Y = \{y_1, y_2\}$  be two sets. Consider the fuzzy sets  $\mu_1, \mu_2, \mu_3$  on  $X$  and  $\nu_1, \nu_2, \nu_3$  on  $Y$  which are defined as shown in the two tables below.

	$x_1$	$x_2$	$x_3$			$y_1$	$y_2$
$\mu_1$	1.0	0.6	0.2		$\nu_1$	1.0	0.4
$\mu_2$	0.0	0.8	1.0		$\nu_2$	0.6	1.0
$\mu_3$	0.9	0.1	0.0		$\nu_3$	0.9	0.5

- a) Show that the system consisting of the two relational equations  $\mu_1 \circ \varrho = \nu_1$  and  $\mu_2 \circ \varrho = \nu_2$  has a solution. Find the greatest solution of this system.
- b) Is the fuzzy relation that can be computed as the union (maximum) of the two Cartesian products  $\mu_1 \otimes \nu_1$  and  $\mu_2 \otimes \nu_2$  also a solution of the system of relational equations considered in a)?
- c) Show that the system consisting of the three relational equations  $\mu_i \circ \varrho = \nu_i$ ,  $i = 1, 2, 3$ , does not have any solution.

## Fuzzy Systems

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### Assignment 30 Fuzzy Control based on Relational Equations

Let  $X = \{1, 2, 3\}$  and  $Y = \{10, 20, 30\}$  be two sets,  $\mu_1, \mu_2$  fuzzy sets on  $X$ , and  $\nu_1, \nu_2$  fuzzy sets on  $Y$ , which are defined as shown in the two tables below.

	1	2	3		10	20	30
$\mu_1$	0.0	0.5	1.0	$\nu_1$	0.0	0.6	1.0
$\mu_2$	1.0	0.4	0.0	$\nu_2$	1.0	0.3	0.0

Consider a fuzzy controller with the following rule base:

**if  $x$  is  $\mu_1$  then  $y$  is  $\nu_1$ ,**

**if  $x$  is  $\mu_2$  then  $y$  is  $\nu_2$ .**

Use the Gödel relation to determine the fuzzy output of this controller for the fuzzy input  $(1 : 0.1, 2 : 1, 3 : 0)$ .