Fuzzy Systems
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Assignment Sheet 10

Assignment 35 Quantifiers

Assuming the same conditions as in Assignment 26: To describe the concept "x is a small number", let $x \in \mathbb{N} \cup \{0\}$ and two membership functions $\mu_1(x)$ and $\mu_2(x)$ be defined as follows:

$$\mu_1(x) = \begin{cases} \frac{20-x}{20}, & \text{if } x < 20 \\ 0, & \text{otherwise} \end{cases}$$
$$\mu_2(x) = 0.95^x$$

What fuzzy truth values do you get for the proposition "There exists a small single-digit prime number in the decimal number system"? Which problem arises thereby?

Hint: Directly use the dual t-conorms $\bot_{\max}(a,b) = \max\{a,b\}$ and $\bot_{\text{sum}}(a,b) = a + b - a \cdot b$.

Assignment 36 Takagi-Sugeno Controller

Construct a Takagi-Sugeno controller with two inputs and one output that computes the following (partially defined) function (cf. Assignment 33):

$$\begin{array}{ll} (1,0) \mapsto 2, & (1,3) \mapsto 4, \\ (0,2) \mapsto 2, & (2,2) \mapsto 4, \\ (2,0) \mapsto 2. & \end{array}$$

Determine the output of your controller for the inputs $(1,1)$ and $(1.5,1.5)$.

Assignment 37 Takagi-Sugeno Controller

Consider the following definition of triangular fuzzy numbers

$$\mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \le x \le m, \\ \frac{r-x}{r-m} & \text{if } m \le x \le r, \\ 0 & \text{otherwise} \end{cases}$$

whereas $l, m, r \in \mathbb{R}$ and $l < m < r$. Now, let a Takagi-Sugeno controller with the rule base be given as follows

R_1 **: if** x **is** μ_1 **then** $y = 2$**,**
R_2 **: if** x **is** μ_2 **then** $y = x$**,**
R_3 **: if** x **is** μ_3 **then** $y = 3 - x^2$**,**

whereas $x \in X = [0,8]$ and X is partitioned by $\mu_1 = \mu_{0,2,4},\ \mu_2 = \mu_{2,4,6},\ \mu_3 = \mu_{4,6,8}$.

a) Compute the output of the controller by using the weighted sum

$$f(x) = \frac{\sum_{r=1}^{3} \mu_{R_r}(x) \cdot f_{R_r}(x)}{\sum_{r=1}^{3} \mu_{R_r}(x)},$$

whereas $\mu_{R_r}(x)$ is the degree of fulfillment that the rule R_r "fires", and f_{R_r} is the output of the rule R_r.

b) Draw the output into a diagram.