Assignment Sheet 10

Assignment 35   Quantifiers

Assuming the same conditions as in Assignment 26: To describe the concept "x is a small number", let $x \in \mathbb{N} \cup \{0\}$ and two membership functions $\mu_1(x)$ and $\mu_2(x)$ be defined as follows:

$$\mu_1(x) = \begin{cases} \frac{20-x}{20}, & \text{if } x < 20 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_2(x) = 0.95^x$$

What fuzzy truth values do you get for the proposition "There exists a small single-digit prime number in the decimal number system"? Which problem arises thereby?

Hint: Directly use the dual t-conorms $\perp_{\max}(a, b) = \max\{a, b\}$ and $\perp_{\text{sum}}(a, b) = a + b - a \cdot b$.

Assignment 36   Takagi-Sugeno Controller

Construct a Takagi-Sugeno controller with two inputs and one output that computes the following (partially defined) function (cf. Assignment 33):

$$\begin{aligned} (1, 0) &\mapsto 2, \quad (1, 3) \mapsto 4, \\ (0, 2) &\mapsto 2, \quad (2, 2) \mapsto 4, \\ (2, 0) &\mapsto 2. \end{aligned}$$

Determine the output of your controller for the inputs $(1, 1)$ and $(1.5, 1.5)$.

Assignment 37   Takagi-Sugeno Controller

Consider the following definition of triangular fuzzy numbers

$$\mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \leq x \leq m, \\ \frac{r-x}{r-m} & \text{if } m \leq x \leq r, \\ 0 & \text{otherwise} \end{cases}$$

whereas $l, m, r \in \mathbb{R}$ and $l < m < r$. Now, let a Takagi-Sugeno controller with the rule base be given as follows

R_1 : **if** x **is** μ_1 **then** $y = 2$,
R_2 : **if** x **is** μ_2 **then** $y = x$,
R_3 : **if** x **is** μ_3 **then** $y = 3 - x^2$,

whereas $x \in X = [0, 8]$ and X is partitioned by $\mu_1 = \mu_{0,2,4}$, $\mu_2 = \mu_{2,4,6}$, $\mu_3 = \mu_{4,6,8}$.

a) Compute the output of the controller by using the weighted sum

$$f(x) = \frac{\sum_{r=1}^{3} \mu_{R_r}(x) \cdot f_{R_r}(x)}{\sum_{r=1}^{3} \mu_{R_r}(x)},$$

whereas $\mu_{R_r}(x)$ is the degree of fulfillment that the rule R_r "fires", and f_{R_r} is the output of the rule R_r.

b) Draw the output into a diagram.

Assignment 38   Fuzzy Clustering

Consider the one-dimensional data set

$$1, 3, 4, 5, 8, 10, 11, 12.$$

We want to process this data set with fuzzy c-means clustering using $c = 2$ (two clusters) and the fuzzifier $m = 2$. Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, *i.e.*

a) Compute the membership degrees of the data points for the initial cluster centers.

b) Compute new cluster centers from the membership degrees that have been obtained before.