

Exercise Sheet 8

Exercise 28 Fuzzy Clustering

Consider the one-dimensional data set

1, 3, 4, 5, 8, 10, 11, 12.

We want to process this data set with fuzzy c -means clustering with $c = 2$ (two clusters) and a fuzzifier of $w = 2$. Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, i.e.:

- a) Compute the membership degrees of the data points for the initial cluster centers!
- b) Compute new cluster centers from the membership degrees computed in this way!

Exercise 29 Expectation Maximization

Consider again the one-dimensional data set used in exercise 28, which we want to process in this exercise with the expectation maximization algorithm to estimate the parameters of a mixture of two normal/Gaussian distributions. Let the prior probabilities of the two clusters be fixed to $\theta_i = \frac{1}{2}$ and the variances to $\sigma_i^2 = 1$, $i = 1, 2$. (That is, only the expected values of the normal distributions — the cluster centers — are to be adapted.) Use the same values for the initial cluster centers as in exercise 28, that is, 1 and 5. Compute one expectation step and one maximization step, i.e.:

- a) Compute the posterior probabilities of the data points for the initial cluster centers!
- b) Estimate new cluster centers from the data point weights computed in this way!

Exercise 30 Fuzzy Clustering

Consider the objective function of fuzzy clustering with a fuzzifier $w = 1$, that is,

$$J(\mathbf{X}, \mathbf{B}, \mathbf{U}) = \sum_{i=1}^c \sum_{j=1}^n u_{ij} d^2(\beta_i, \vec{x}_j),$$

which is to be minimized under the constraint

$$\forall j \in \{1, \dots, n\} : \sum_{i=1}^c u_{ij} = 1.$$

Show that one obtains a hard/crisp assignment of the data points even if the membership degrees u_{ij} may come from the interval $[0, 1]$. That is, show that for the minimum of the objective function J it is $\forall i \in \{1, \dots, c\} : \forall j \in \{1, \dots, n\} : u_{ij} \in \{0, 1\}$. (Hint: You may find it easier to consider the special case $c = 2$ (two clusters) and to examine the term for a single data point \vec{x}_j . Then generalize the result.)

Exercise 31 Agglomerative Clustering

Let the following one-dimensional data set be given:

$$2, 5, 11, 12, 17, 21, 32.$$

Process this data set with hierarchical agglomerative clustering using

- a) the centroid method,
- b) the single linkage method,
- c) the complete linkage method!

Draw a dendrogram for each case!

Additional Exercise Fuzzy Clustering

Consider the objective function

$$J(\mathbf{X}, \mathbf{B}, \mathbf{U}) = \sum_{i=1}^c \sum_{j=1}^n u_{ij} d^2(\beta_i, \vec{x}_j),$$

which is to be minimized under the constraint

$$\forall j \in \{1, \dots, n\} : \sum_{i=1}^c \sqrt{u_{ij}} = 1.$$

Derive the update formulae for the membership degrees and the cluster centers using the Euclidean distance. How does the result differ from standard fuzzy clustering with a fuzzifier $w = 2$? (In particular, consider the cluster centers.)