Intelligent Data Analysis | Summer 2016
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Exercise Sheet 4

Exercise 13 Estimators

Let $X = (X_1, \ldots, X_n)$ be the random vector underlying a random sample of size n. We assume that the random variables X_i are independent and identically distributed according to the exponential distribution $f_X(x;\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}$, $x > 0$. We desire to estimate the parameter θ of this distribution. The most commonly used estimator for θ is $W_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$. Here, however, we consider the estimator $W_2 = n \cdot X_{\min} = n \min_{i=1}^{n} X_i$. Determine the probability density function of this estimator, that is, $f_{W_2}(w;\theta)$.
Hint: Recall the technical trick to consider the complementary event instead of the event itself, which we already used in exercise 1.

Exercise 14 Unbiasedness of Estimators

a) Let W_1 and W_2 be two unbiased estimators for the unknown parameter θ. If we want $W = aW_1 + bW_2$ to be an unbiased estimator for θ as well, what conditions must hold for a and b?
b) Show: if we desire to estimate the parameter $\mu = E(X)$, then $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is always an unbiased estimator for μ.

Exercise 15 Maximum Likelihood Estimation

Determine a maximum likelihood estimator for the parameter θ of a uniform distribution on the interval $[0, \theta]$! Reminder: the random variables underlying the sample vector have the probability density function

$$f_X(x;\theta) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{\theta}, & \text{if } 0 \le x \le \theta, \\ 0, & \text{if } x > \theta. \end{cases}$$

Check whether the resulting estimator is consistent and unbiased!
(Hint: When maximing the likelihood function bear in mind that certain frame conditions have to be satisfied. Compare your result to the estimators discussed in the lecture.)

Exercise 16 Confidence Intervals

In the year 1972 45195 of the 87827 live births in Lower Saxony were boys. From this data, determine a point estimator for the unknown probability p that a newly born child is a boy, as well as confidence intervals for the confidence levels

a) $\alpha = 0.01$ (99% confidence interval) and
b) $\alpha = 0.001$ (99.9% confidence interval).

(Hint: The needed quantiles of the normal distribution may be computed with the C program `ndqtl.c`, which is available on the lecture's WWW page. Quantile: argument value corresponding to a given function value of a distribution function; analogous to the quantiles of a sample.)