
Chapter 8:

Neuro-Fuzzy Systems

Neuro-Fuzzy Systems

- Building a fuzzy system requires
 - prior knowledge (fuzzy rules, fuzzy sets)
 - manual tuning: *time consuming and error-prone*
- Therefore: Support this process by learning
 - learning fuzzy rules (structure learning)
 - learning fuzzy set (parameter learning)

Approaches from Neural Networks can be used

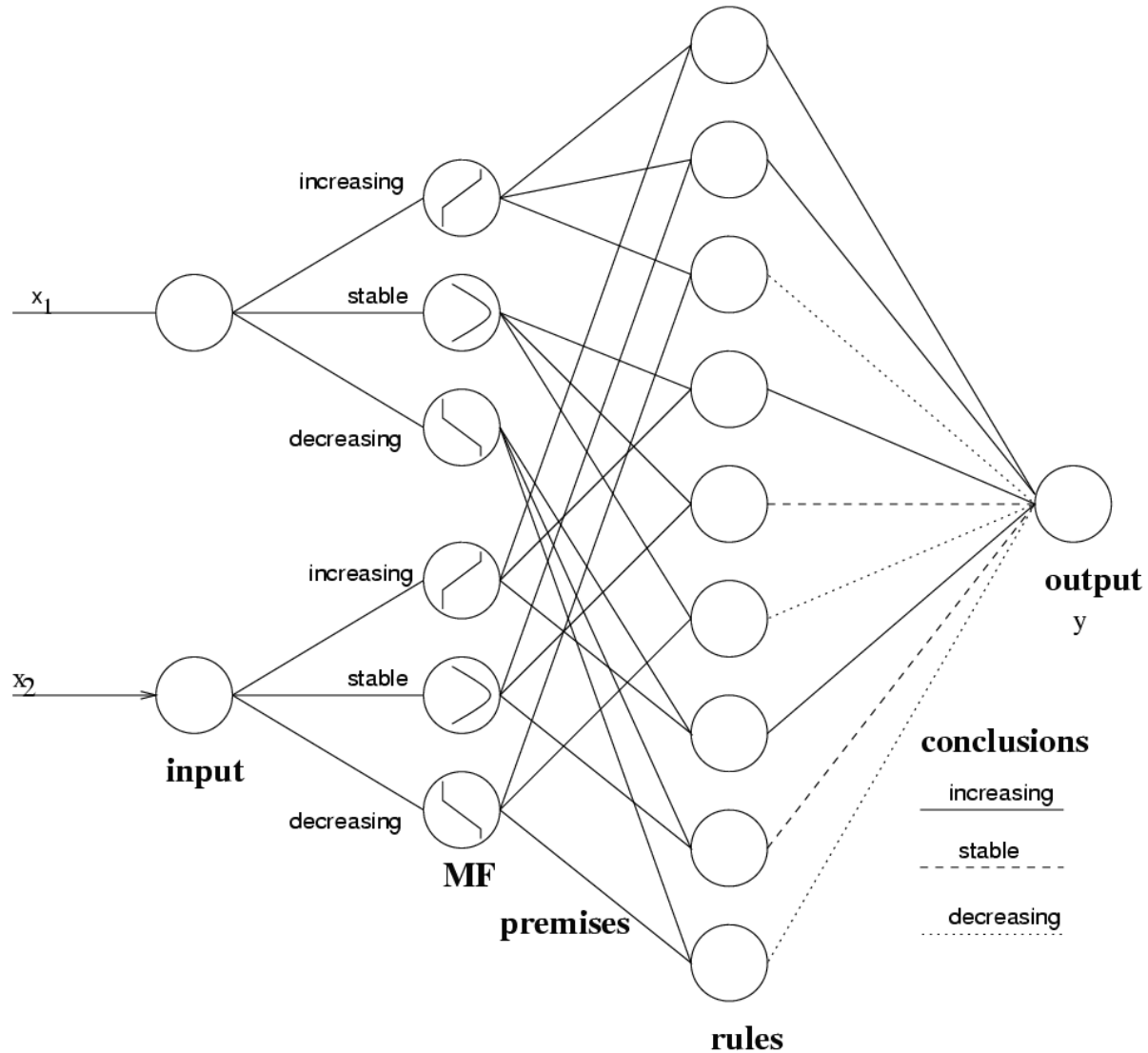
■ Database: time series from 1986 - 1997

DAX	Composite DAX
German 3 month interest rates	Return Germany
Morgan Stanley index Germany	Dow Jones industrial index
DM / US-\$	US treasury bonds
Gold price	Nikkei index Japan
Morgan Stanley index Europe	Price earning ratio

Fuzzy Rules in Finance

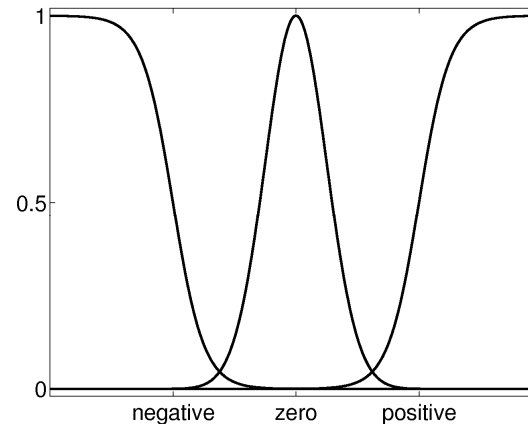
- Trend Rule
 - IF DAX = decreasing AND US-\$ = decreasing
 - THEN DAX prediction = decrease
 - WITH high certainty
- Turning Point Rule
 - IF DAX = decreasing AND US-\$ = increasing
 - THEN DAX prediction = increase
 - WITH low certainty
- Delay Rule
 - IF DAX = stable AND US-\$ = decreasing
 - THEN DAX prediction = decrease
 - WITH very high certainty
- In general
 - IF x_1 is μ_1 AND x_2 is μ_2
 - THEN $y = \eta$
 - WITH weight k

Neuro-Fuzzy Architecture



From Rules to Neural Networks

1. Evaluation of membership degrees



2. Evaluation of rules (rule activity)

$$\mu_l: \mathbb{R}^n \rightarrow [0,1]^r, \quad \underline{x} \Rightarrow \prod_{j=1}^{D_l} \mu_{c,s}^{(j)}(x_i)$$

3. Accumulation of rule inputs and normalization

$$\text{NF}: \mathbb{R}^n \rightarrow \mathbb{R}, \quad \underline{x} \Rightarrow \sum_{l=1}^r w_l \frac{k_l \mu_l(\underline{x})}{\sum_{j=1}^r k_j \mu_j(\underline{x})}$$

Reduction of the dimension of the weight space

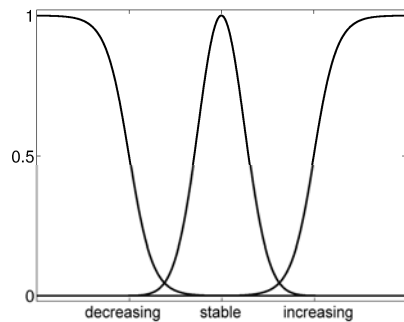
1. Membership functions of different inputs share their parameters,

e.g.

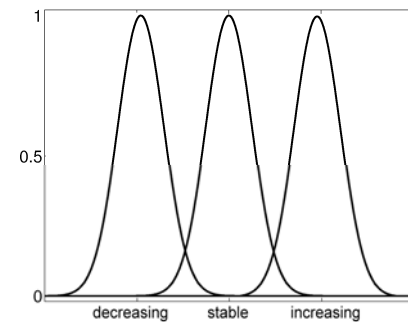
$$\mu_{dax}^{stable} \equiv \mu_{cdax}^{stable}$$

2. Membership functions of the same input variable are not allowed to pass each other, they must keep their original order,

e.g.



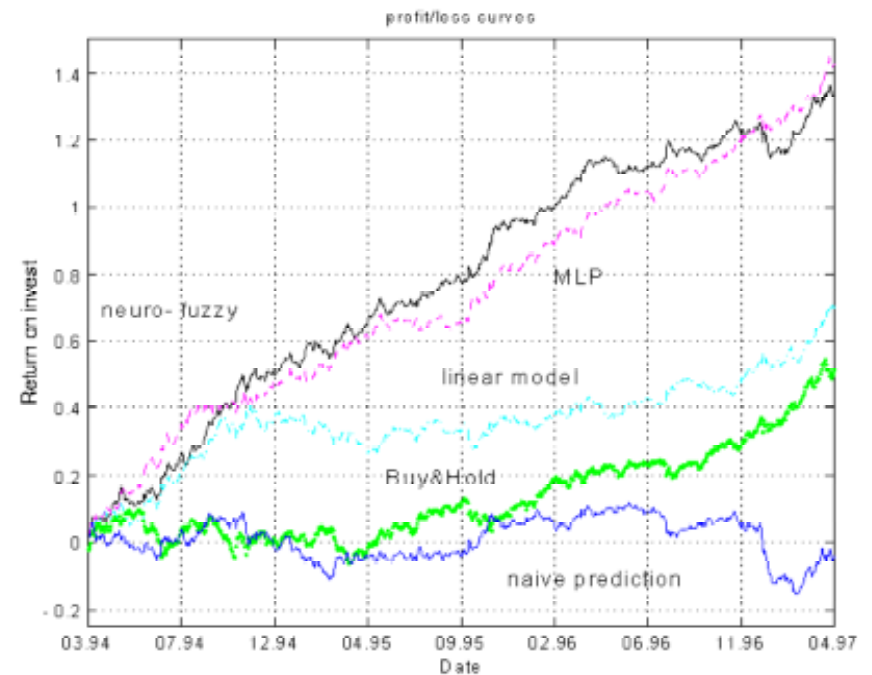
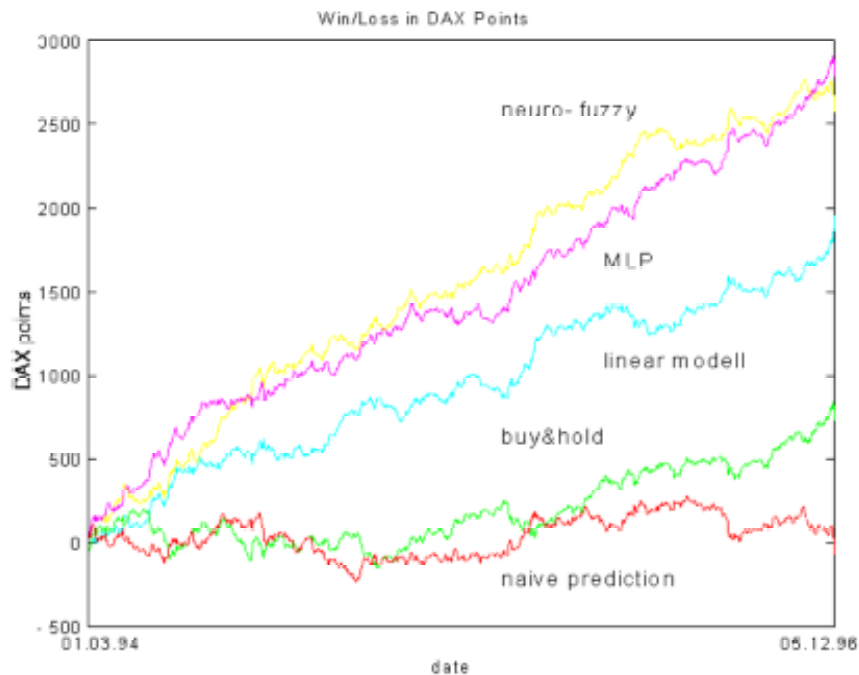
$$\mu^{decreasing} < \mu^{stable} < \mu^{increasing}$$



- Benefits:
- the optimized rule base can still be interpreted
 - the number of free parameters is reduced

Return-on-Investment Curves of the Different Models

Validation data from March 01, 1994 until April 1997



■ Neuro-Fuzzy System:

- System of linguistic rules (fuzzy rules).
- Not rules in a logical sense, but function approximation.
- Fuzzy rule = vague prototype / sample.

■ Neuro-Fuzzy-System:

- Adding a learning algorithm inspired by neural networks.
- Feature: local adaptation of parameters.

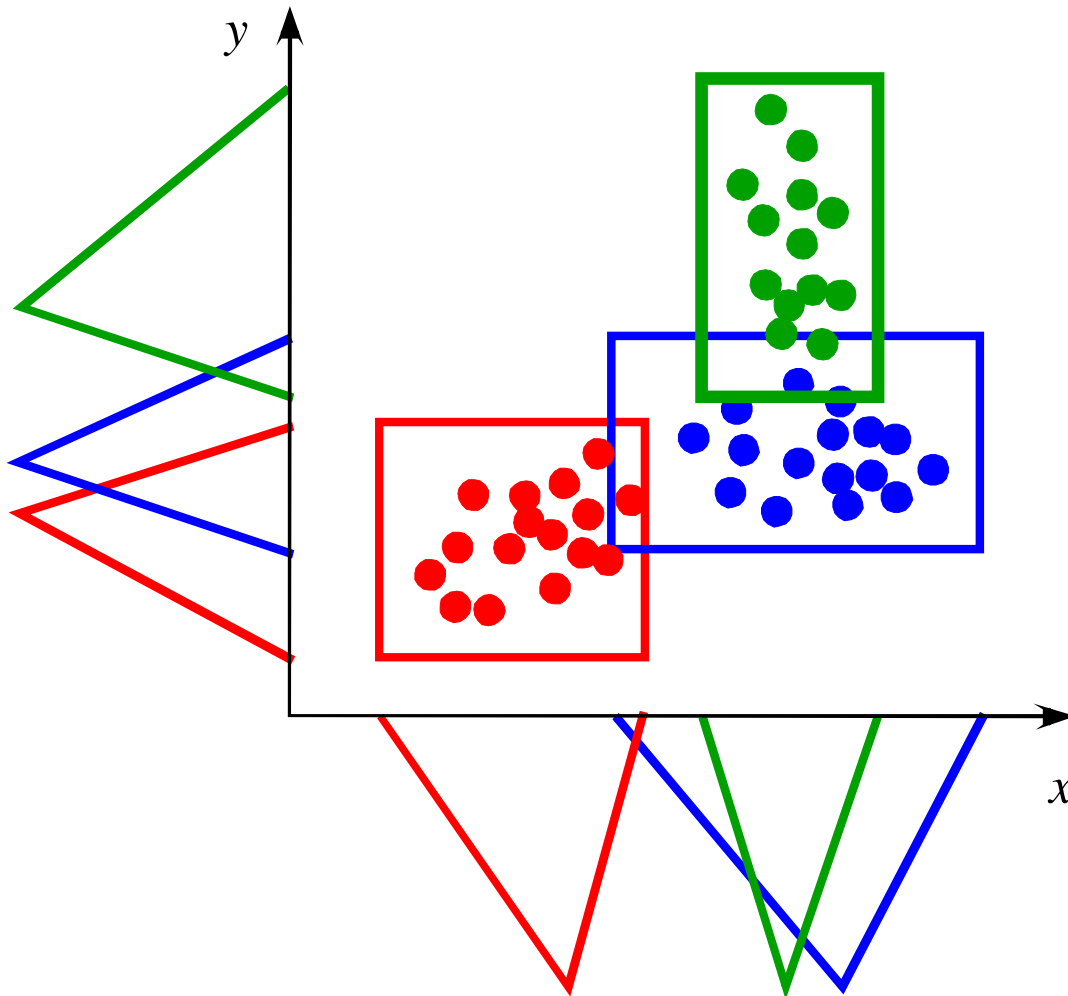
A Neuro-Fuzzy System

- is a fuzzy system trained by heuristic learning techniques derived from neural networks
- can be viewed as a 3-layer neural network with fuzzy weights and special activation functions
- is always interpretable as a fuzzy system
- uses constraint learning procedures
- is a function approximator (classifier, controller)

Learning Fuzzy Rules

- Cluster-oriented approaches
=> find clusters in data, each cluster is a rule
- Hyperbox-oriented approaches
=> find clusters in the form of hyperboxes
- Structure-oriented approaches
=> used predefined fuzzy sets to structure the data space, pick rules from grid cells

Hyperbox-Oriented Rule Learning



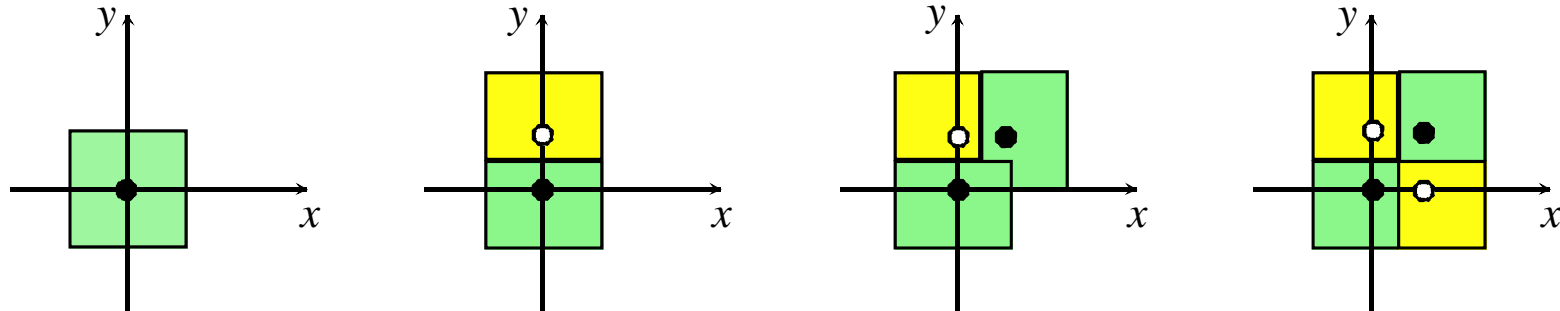
Search for hyperboxes
in the data space

Create fuzzy rules by
projecting the
hyperboxes

Fuzzy rules and fuzzy
sets are created at the
same time

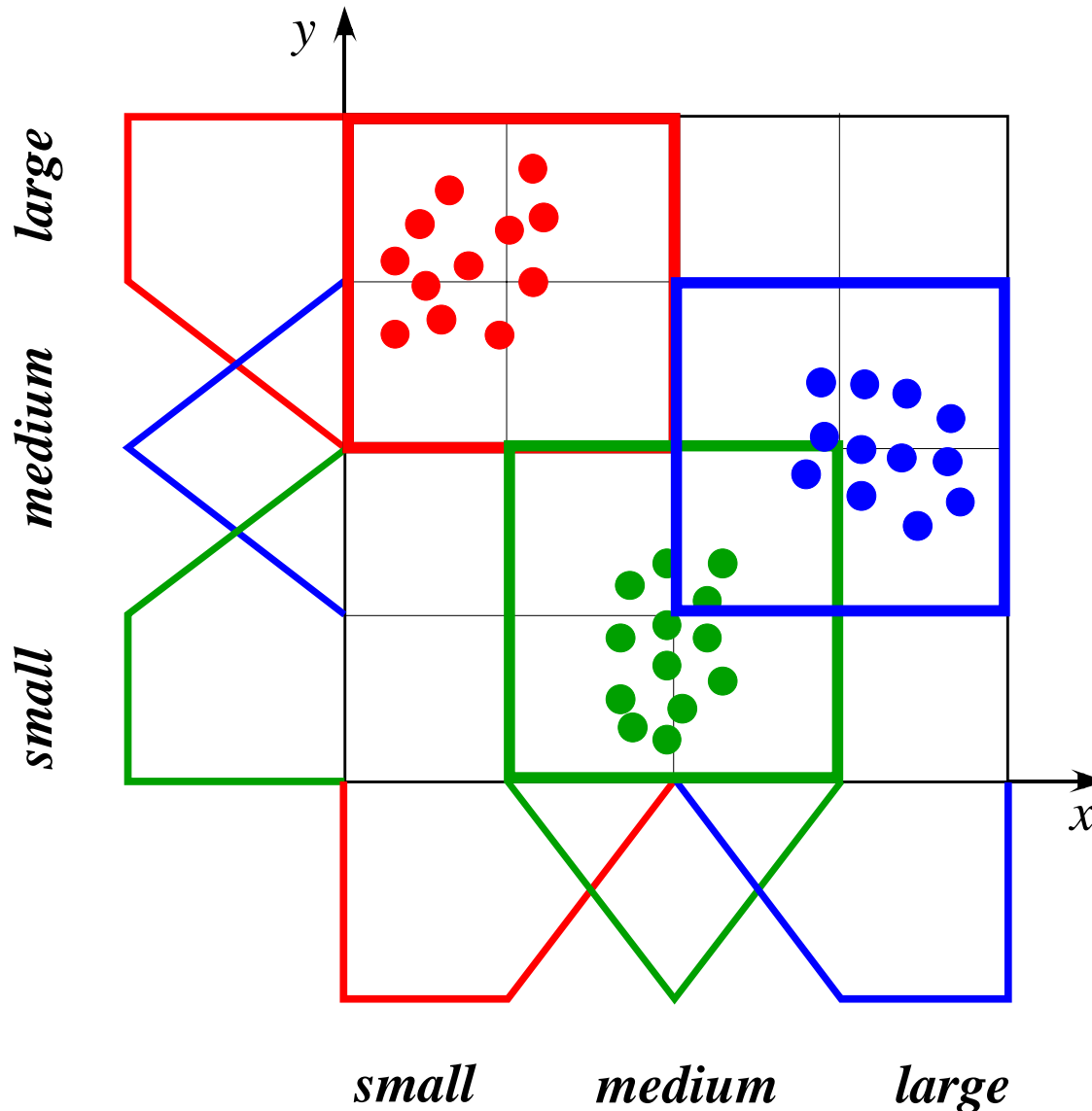
Usually very fast

Hyperbox-Oriented Rule Learning



- Detect hyperboxes in the data, example: XOR function
- Advantage over fuzzy cluster analysis:
 - No loss of information when hyperboxes are represented as fuzzy rules
 - Not all variables need to be used, don't care variables can be discovered
- Disadvantage: each fuzzy rules uses individual fuzzy sets, i.e. the rule base is complex.

Structure-Oriented Rule Learning



Provide initial fuzzy sets for all variables.

The data space is partitioned by a fuzzy grid

Detect all grid cells that contain data (approach by Wang/Mendel 1992)

Compute best consequents and select best rules (extension by Nauck/Kruse 1995, NEFCLASS model)

Structure-Oriented Rule Learning

- Simple: Rule base available after two cycles through the training data
 - 1. Cycle: discover all antecedents
 - 2. Cycle: determine best consequents

- Missing values can be handled

- Numeric and symbolic attributes can be processed at the same time (mixed fuzzy rules)

- Advantage: All rules share the same fuzzy sets

- Disadvantage: Fuzzy sets must be given

Learning Fuzzy Sets

- Gradient descent procedures
only applicable, if differentiation is possible, e.g. for Sugeno-type fuzzy systems.
- Special heuristic procedures that do not use gradient information.
- The learning algorithms are based on the idea of backpropagation.

Learning Fuzzy Sets: Constraints

- Mandatory constraints:
 - Fuzzy sets must stay normal and convex
 - Fuzzy sets must not exchange their relative positions (they must not „pass“ each other)
 - Fuzzy sets must always overlap
- Optional constraints
 - Fuzzy sets must stay symmetric
 - Degrees of membership must add up to 1.0
- The learning algorithm must enforce these constraints.

Example: Medical Diagnosis

- Results from patients tested for breast cancer (Wisconsin Breast Cancer Data).
- Decision support: Do the data indicate a malignant or a benign case?
- A surgeon must be able to check the classification for plausibility.
- We are looking for a simple and interpretable classifier:
⇒ **knowledge discovery.**

Example: WBC Data Set

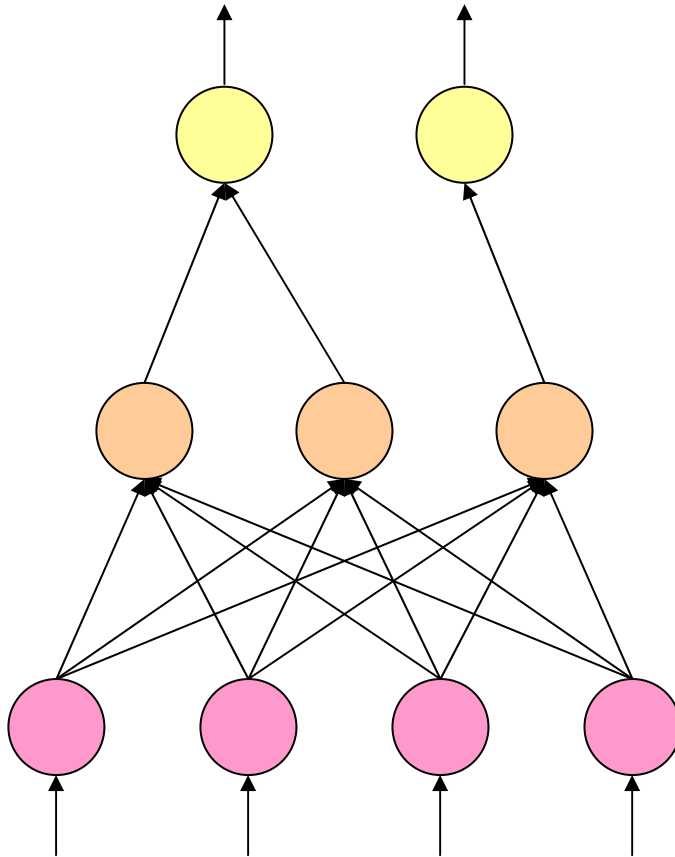
- 699 cases (16 cases have missing values).
- 2 classes: benign (458), malignant (241).
- 9 attributes with values from $\{1, \dots, 10\}$
(ordinal scale, but usually interpreted as a numerical scale).
- Experiment: x_3 and x_6 are interpreted as nominal attributes.
- x_3 and x_6 are usually seen as „important“ attributes.

Applying NEFCLASS-J

- Tool for developing Neuro-Fuzzy Classifiers
- Written in JAVA
- Free version for research available
- Project started at Neuro-Fuzzy Group of University of Magdeburg, Germany

The screenshot displays the NEFCLASS-J software interface. The main window shows a 'Fuzzy Set Learning' plot with 'Error Misclassifications' on the y-axis (0.00 to 49.00) and an x-axis (0.00 to 180.00). Below it, the 'Fuzzy Sets' window shows three fuzzy sets: 'sm' (small), 'mid' (medium), and 'lg' (large) plotted against a variable range from 0 to 7.0. The 'List of Variables' window lists: sepal length, sepal width, petal length, and petal width. The 'Rule Learning' window shows a log of the learning process, including a warning that the rule base covers only 94% of the training data. The 'Edit Rules' window shows a rule editor with 'Variables' (sepal length, sepal width, petal length, petal width) and 'Fuzzy Sets' (small, medium, large). The 'Antecedent of Rule R0' is 'sepal length is small and sepal width is medium and petal length is small and petal width is small'. The 'Consequent (Class)' is 'Iris Setosa, Iris Versicolor, Iris Virginica'. The 'About' window shows the NEFCLASS-J logo and credits to Ulrike Nauck and Dr. Detlef Nauck.

NEFCLASS: Neuro-Fuzzy Classifier



Output variables (class labels)

Unweighted connections

Fuzzy rules

Fuzzy sets (antecedents)

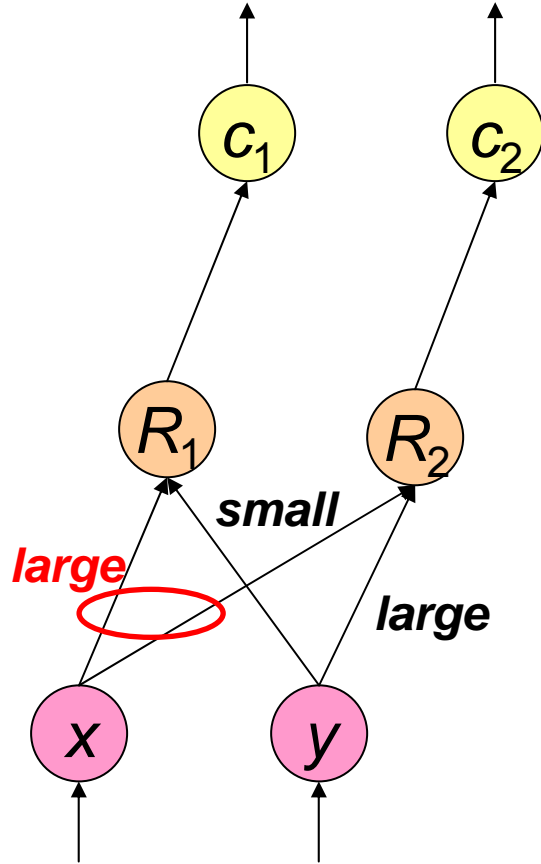
Input variables (attributes)

NEFCLASS: Features

- Automatic induction of a fuzzy rule base from data
- Training of several forms of fuzzy sets
- Processing of numeric and symbolic attributes
- Treatment of missing values (no imputation)
- Automatic pruning strategies
- Fusion of expert knowledge and knowledge obtained from data

Representation of Fuzzy Rules

Example: 2 Rules



R_1 : if x is *large* and y is *small*, then class is c_1 .

R_2 : if x is *large* and y is *large*, then class is c_2 .

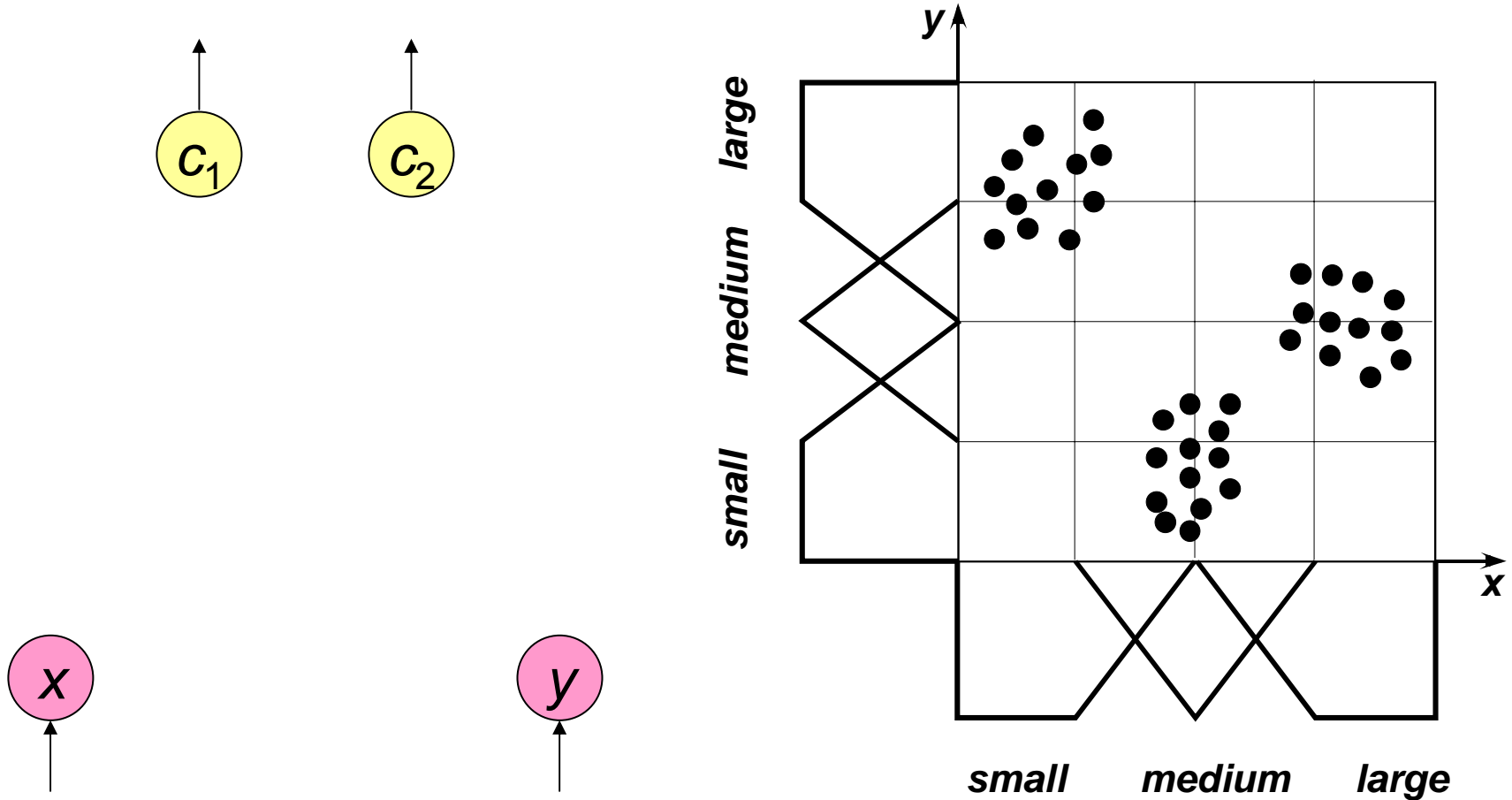
The connections $x \rightarrow R_1$ and $x \rightarrow R_2$ are linked.

The fuzzy set *large* is a shared weight.

That means the term *large* has always the same meaning in both rules.

1. Training Step: Initialisation

Specify initial fuzzy partitions for all input variables



2. Training Step: Rule Base

Algorithm:

```
for (all patterns  $p$ ) do
    find antecedent  $A$ ,
    such that  $A(p)$  is maximal;
    if ( $A \notin L$ ) then add  $A$  to  $L$ ;
end;

for (all antecedents  $A \in L$ ) do
    find best consequent  $C$  for  $A$ ;
    create rule base candidate  $R = (A, C)$ ;
    Determine the performance of  $R$ ;
    Add  $R$  to  $B$ ;
end;

Select a rule base from  $B$ ;
```

Variations:

Fuzzy rule bases can also be created by using prior knowledge, fuzzy cluster analysis, fuzzy decision trees, genetic algorithms, ...

Selection of a Rule Base

Performance of a Rule :

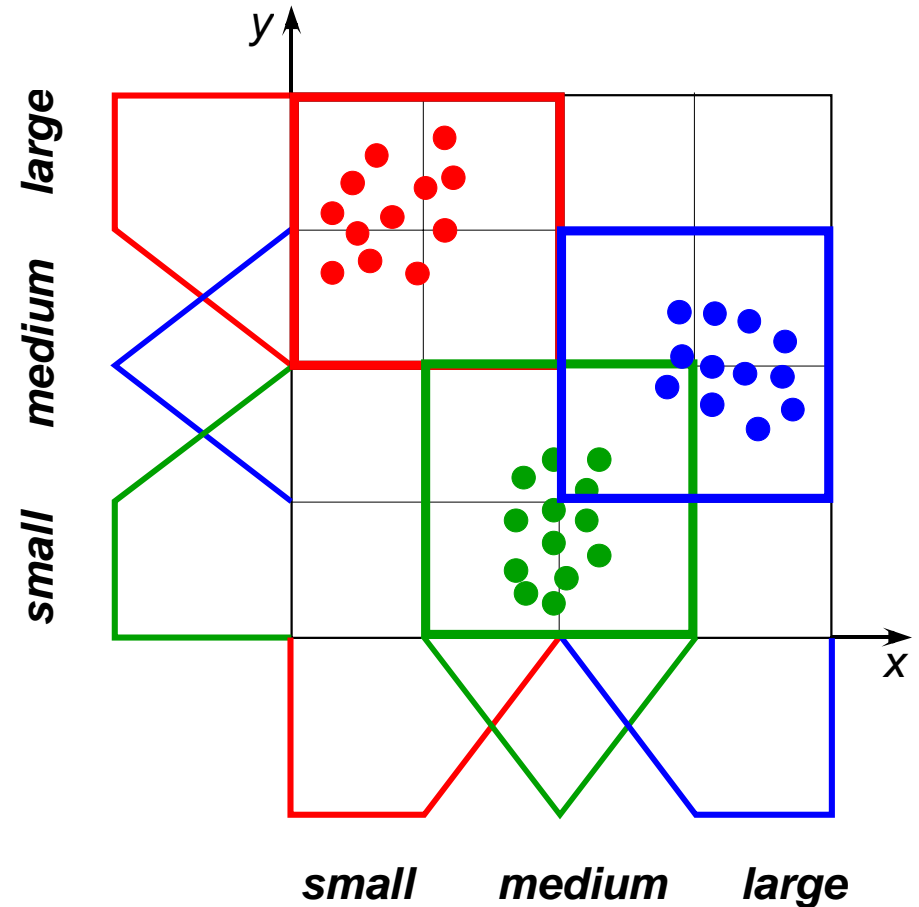
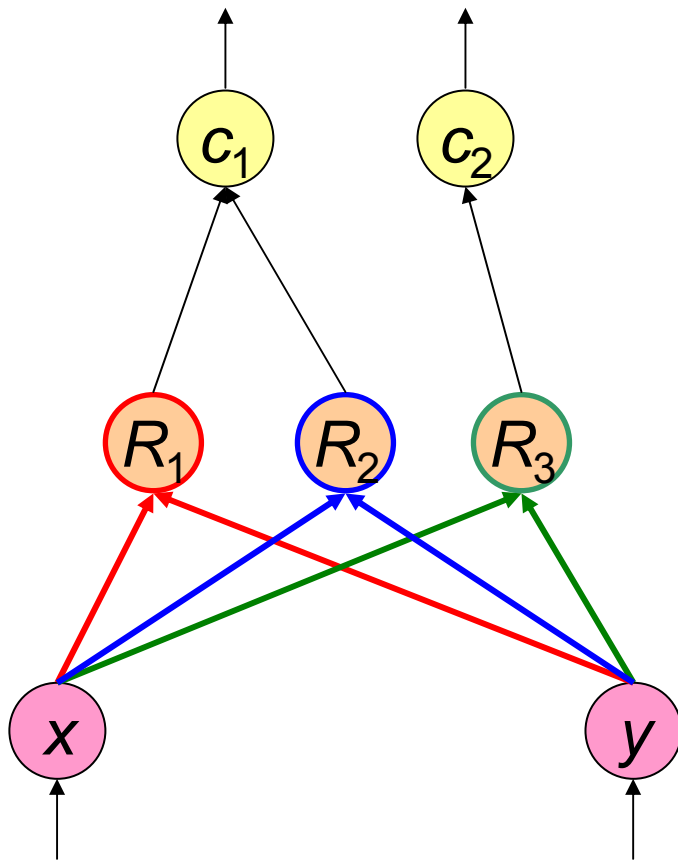
$$P_r = \frac{1}{N} \sum_{p=1}^N (-1)^c R_r(\mathbf{x}_p), \text{ with}$$

$$c = \begin{cases} 0 & \text{if } \text{class}(\mathbf{x}_p) = \text{con}(R_r), \\ 1 & \text{otherwise.} \end{cases}$$

- Order rules by performance.
- Either select the best r rules or the best r/m rules per class.
- r is either given or is determined automatically such that all patterns are covered.

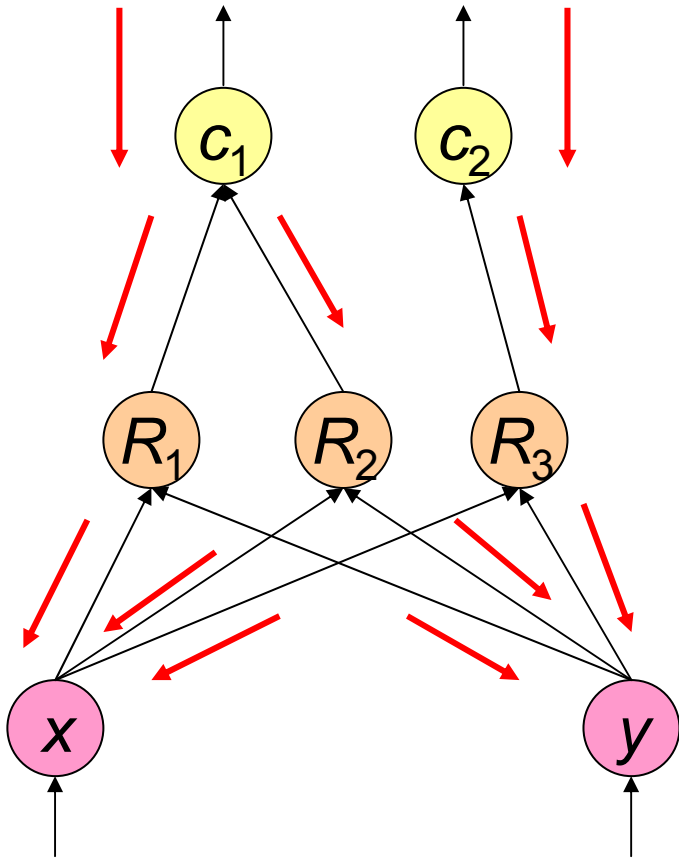
Rule Base Induction

NEFCLASS uses a modified Wang-Mendel procedure



Computing the Error Signal

Error Signal



Fuzzy Error (j th output):

$$E_j = \text{sgn}(d) \cdot (1 - \gamma(d)), \quad \text{with } d = t_j - o_j$$

$$\text{and } \gamma: \mathfrak{R} \rightarrow [0, 1], \quad \gamma(d) = e^{-\left(\frac{a \cdot d}{d_{\max}}\right)^2}$$

(t : correct output, o : actual output)

Rule Error:

$$E_r = (\tau_r (1 - \tau_r) + \varepsilon) E_{\text{con}(R_r)}, \quad \text{with } 0 < \varepsilon \ll 1$$

3. Training Step: Fuzzy Sets

**Example:
triangular
membership
function.**

$$\mu_{a,b,c} : \mathcal{R} \rightarrow [0,1], \mu_{a,b,c}(x) = \left. \begin{array}{ll} \frac{x-a}{b-a} & \text{if } x \in [a,b) \\ \frac{c-x}{c-b} & \text{if } x \in [b,c] \\ 0 & \text{otherwise} \end{array} \right\}$$

**Parameter
updates for an
antecedent
fuzzy set.**

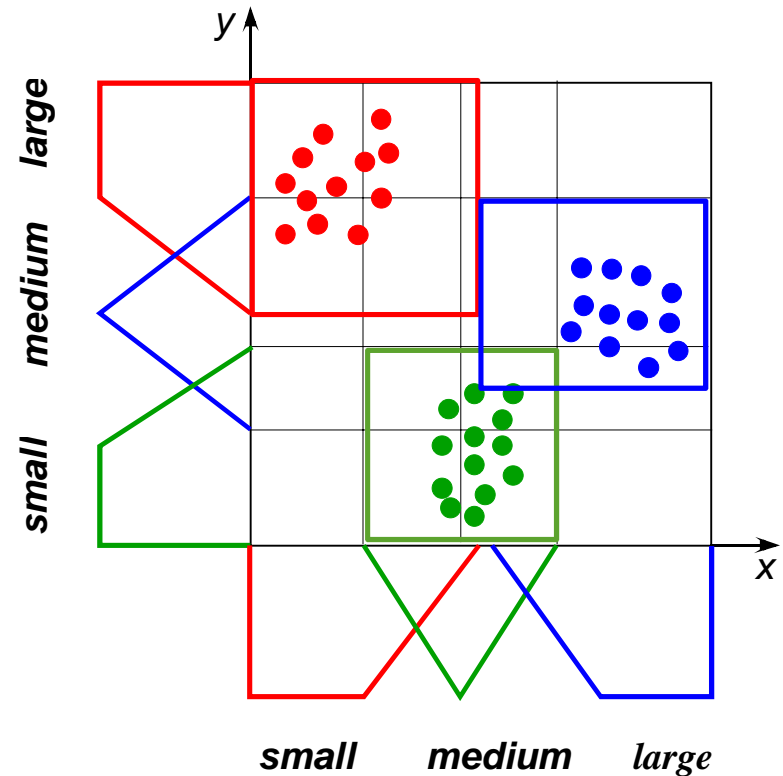
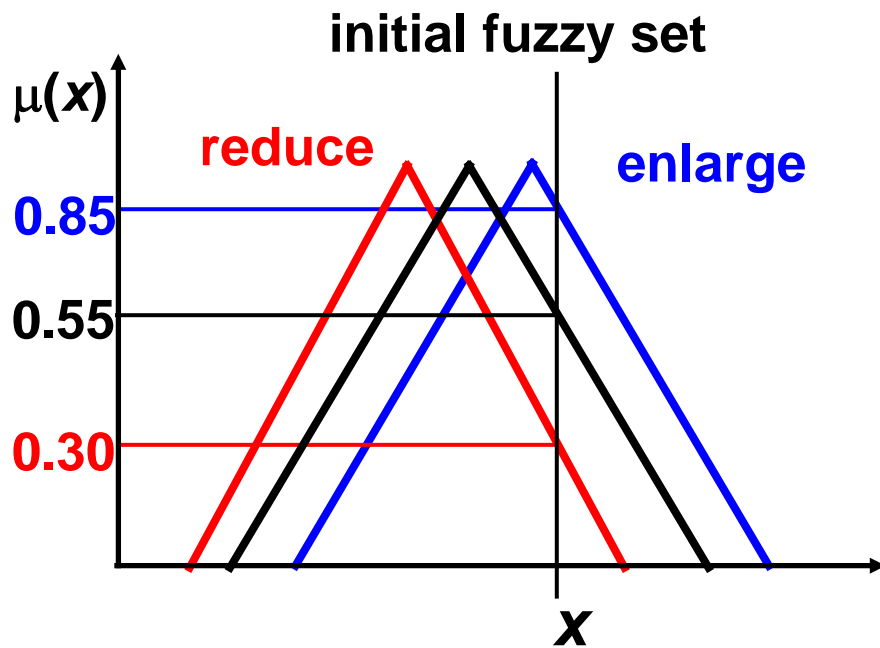
$$f = \begin{cases} \sigma \mu(x) & \text{if } E < 0 \\ \sigma (1 - \mu(x)) & \text{otherwise} \end{cases}$$

$$\Delta b = f \cdot E \cdot (c - a) \cdot \text{sgn}(x - b)$$

$$\Delta a = -f \cdot E \cdot (b - a) + \Delta b$$

$$\Delta c = f \cdot E \cdot (c - b) + \Delta b$$

Training of Fuzzy Sets



Heuristics: a fuzzy set is moved **away from x** (**towards x**) and its support is **reduced** (**enlarged**), in order to **reduce** (**enlarge**) the degree of membership of x .

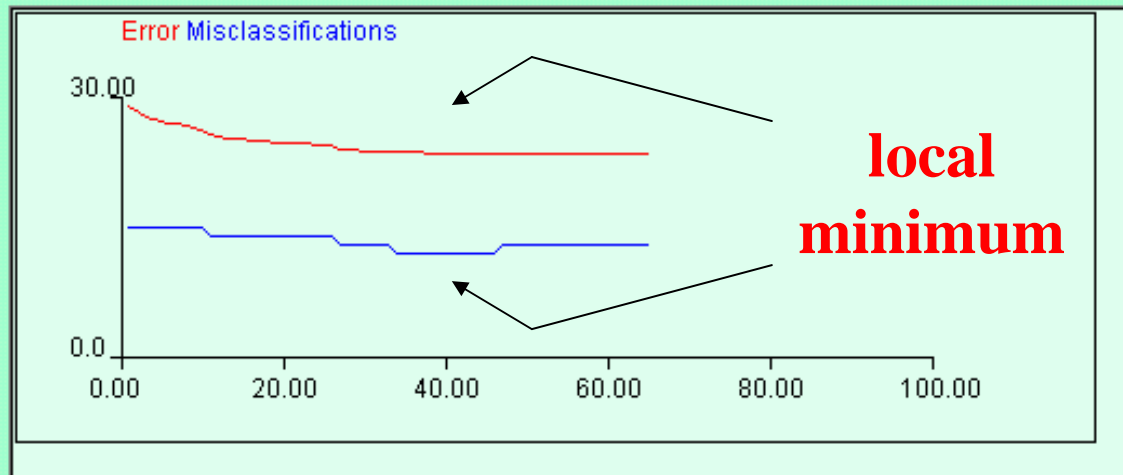
Training of Fuzzy Sets

Algorithm:

```
repeat
  for (all patterns) do
    accumulate parameter updates;
    accumulate error;
  end;
  modify parameters;
until (no change in error);
```

Variations:

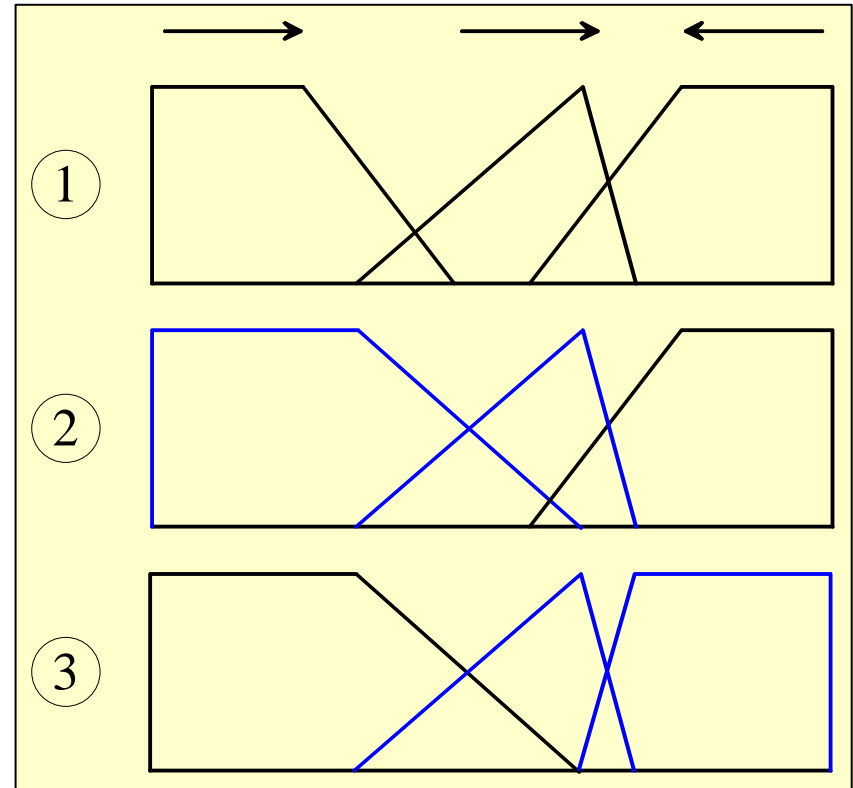
- Adaptive learning rate
- Online-/Batch Learning
- optimistic learning (n step look ahead)



Observing the error on a validation set

Constraints for Training Fuzzy Sets

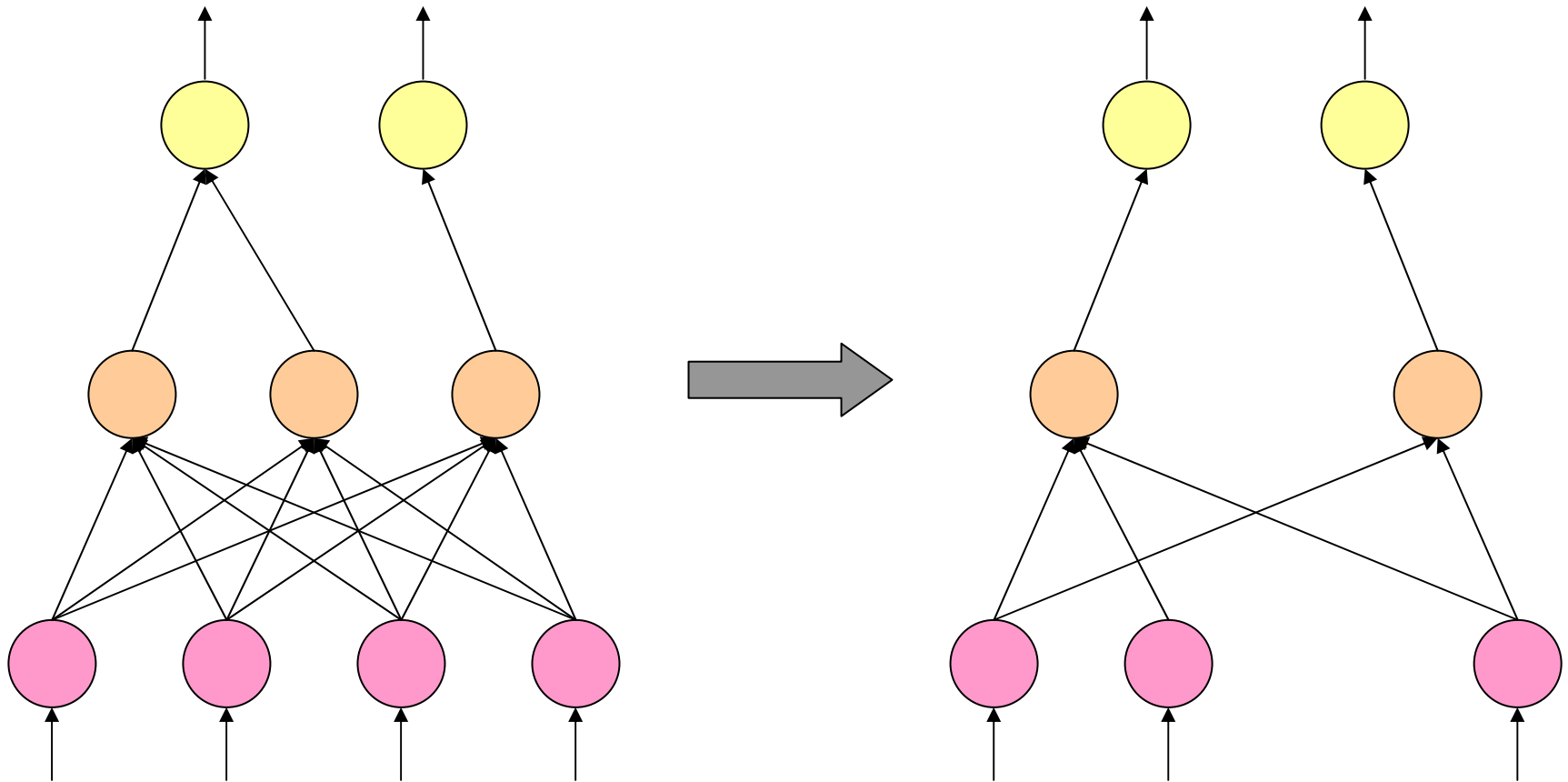
- Valid parameter values
- Non-empty intersection of adjacent fuzzy sets
- Keep relative positions
- Maintain symmetry
- Complete coverage
(degrees of membership add up to 1 for each element)



Correcting a partition after
modifying the parameters

4. Training Step: Pruning

Goal: remove variables, rules and fuzzy sets, in order to improve interpretability and generalisation.



Pruning

Algorithm:

repeat

select pruning method;

repeat

execute pruning step;
train fuzzy sets;

if (no improvement)
then undo step;

until (no improvement);

until (no further method);

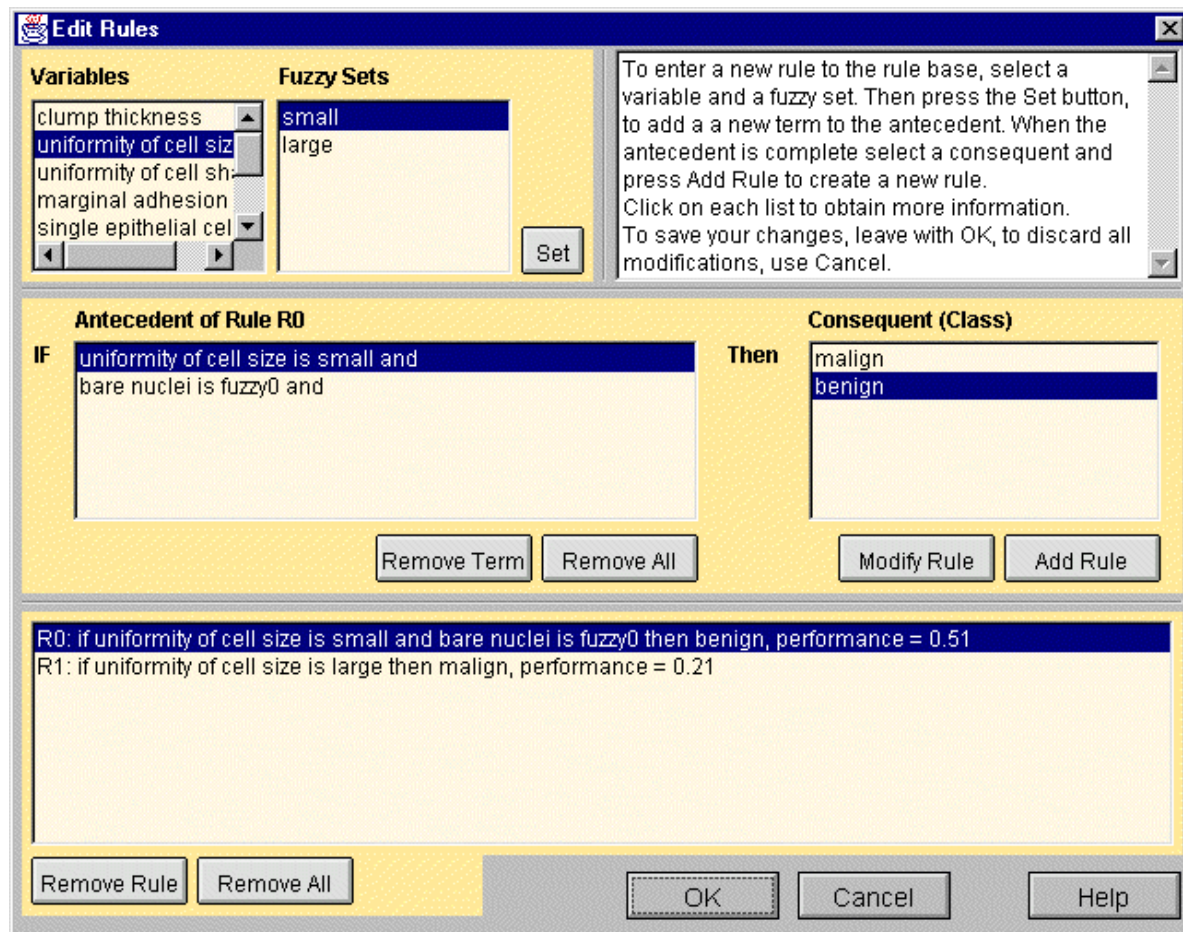
Pruning Methods:

1. Remove variables
(use correlations, information gain etc.)
2. Remove rules
(use rule performance)
3. Remove terms
(use degree of fulfilment)
4. Remove fuzzy sets
(use fuzziness)

WBC Learning Result: Fuzzy Rules

R_1 : if uniformity of cell size is *small* and bare nuclei is fuzzy0 then *benign*

R_2 : if uniformity of cell size is *large* then *malignant*



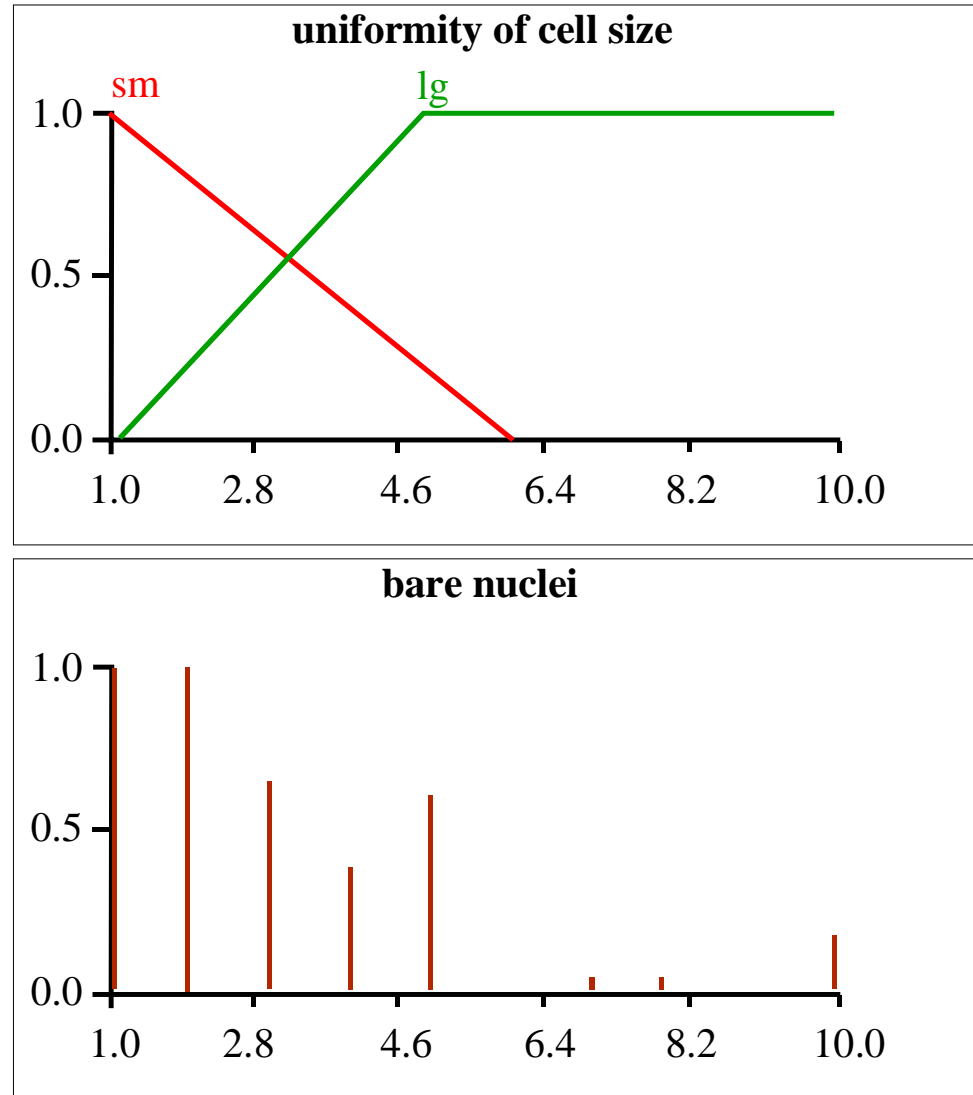
WBC Learning Result: Classification Performance

	Predicted Class							
	malign		benign		not classified		sum	
malign	228	(32.62%)	13	(1.86%)	0	(0%)	241	(34.99%)
benign	15	(2.15%)	443	(63.38%)	0	(0%)	458	(65.01%)
sum	243	(34.76%)	456	(65.24%)	0	(0%)	699	(100.00%)

Estimated Performance on Unseen Data (Cross Validation)

- NEFCLASS-J: 95.42%
- Discriminant Analysis: 96.05%
- C 4.5: 95.10%
- NEFCLASS-J (numeric): 94.14%
- Multilayer Perceptron: 94.82%
- C 4.5 Rules: 95.40%

WBC Learning Result: Fuzzy Sets



NEFCLASS-J

The screenshot displays the NEFCLASS-J software interface with several windows open:

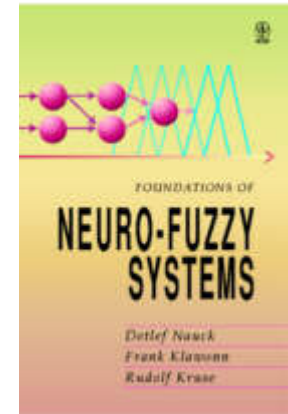
- Fuzzy Set Learning:** A graph showing 'Error Misclassifications' over time. The red line (Error) starts at 49.00 and decreases to approximately 26.46. The blue line (Misclassifications) starts at 0.00 and increases to approximately 15.00.
- NEFCLASS (Main Window):** Shows project status: 'ready', classifier: '5 rules, trained', and training data: 'Iris_new.dat'.
- Fuzzy Sets:** A graph showing three fuzzy sets: 'sm' (small), 'md' (medium), and 'lg' (large) over a range of 1.0 to 7.0. 'sm' is a trapezoidal function, 'md' is a triangular function, and 'lg' is a trapezoidal function.
- Rule Learning:** A text window showing the results of rule learning: '20 possible rules found. Now determine the optimal combination of consequents is complete. Writing the rules to the log file, please wait. Best rule learning: selecting the best 5 rules. Best rules selected, trimming the rule base. Rule base with 5 rules created. WARNING: This rule base covers only 94% of all training data but this may improve during fuzzy set learning. Performance on training data (100.0% of all cases): 150 patterns, 15 misclassifications (error = 26.457628)'.
- Edit Rules:** A window for editing rules. It shows a list of variables (sepal length, sepal width, petal length, petal width) and fuzzy sets (small, medium, large). The 'Antecedent of Rule R0' is 'sepal length is small and sepal width is medium and petal length is small and petal width is small'. The 'Consequent (Class)' is 'Iris Setosa, Iris Versicolor, Iris Virginica'.
- About:** A window showing the NEFCLASS-J logo and version information: 'Version 1.0', 'Programming and GUI Design: Ulrike Nauck', 'NEFCLASS Model and Learning Algorithm: Dr. Detlef Nauck', and '(c) Ulrike Nauck, Braunschweig, 1998'.

Resources

Detlef Nauck, Frank Klawonn & Rudolf Kruse:

Foundations of Neuro-Fuzzy Systems

Wiley, Chichester, 1997, ISBN: 0-471-97151-0



Neuro-Fuzzy Software (NEFCLASS, NEFCON, NEFPROX):

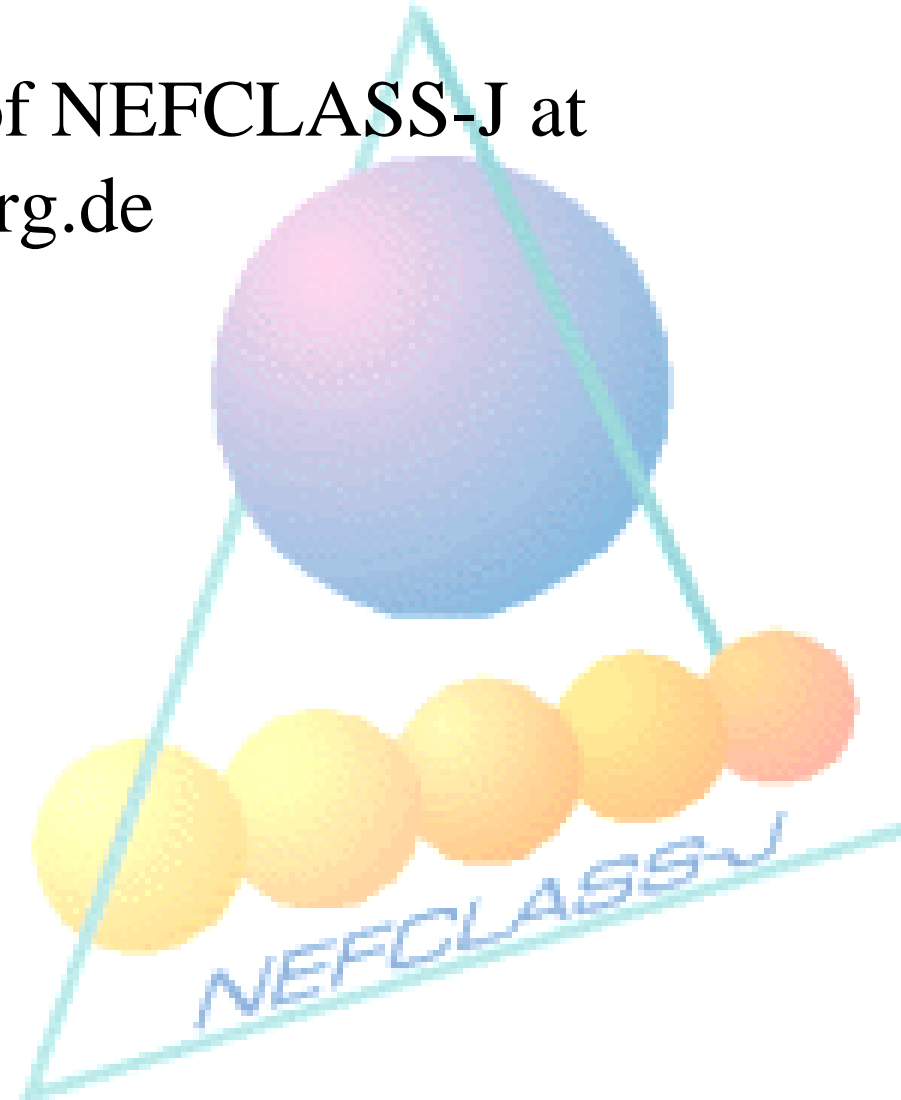
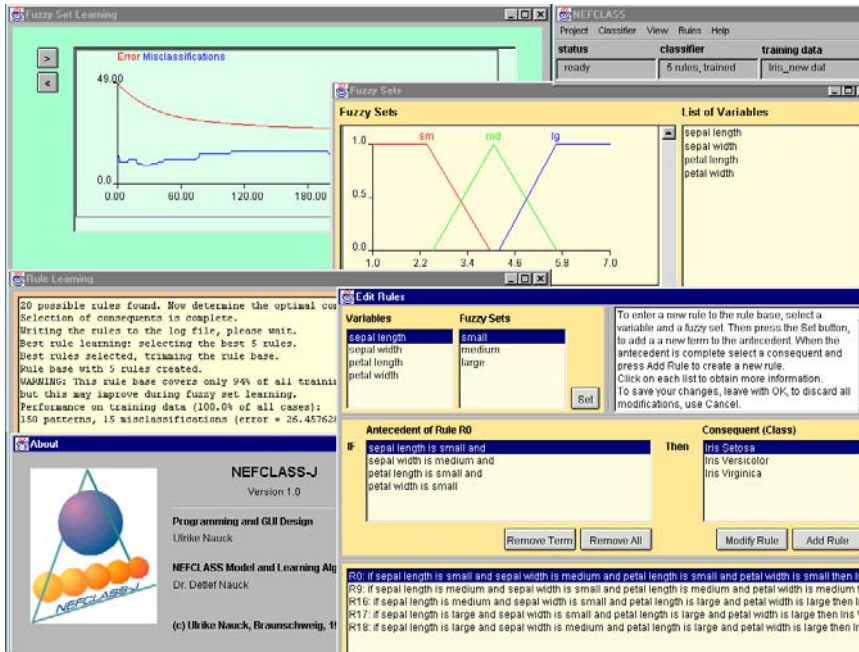
<http://www.neuro-fuzzy.de>

Beta-Version of NEFCLASS-J:

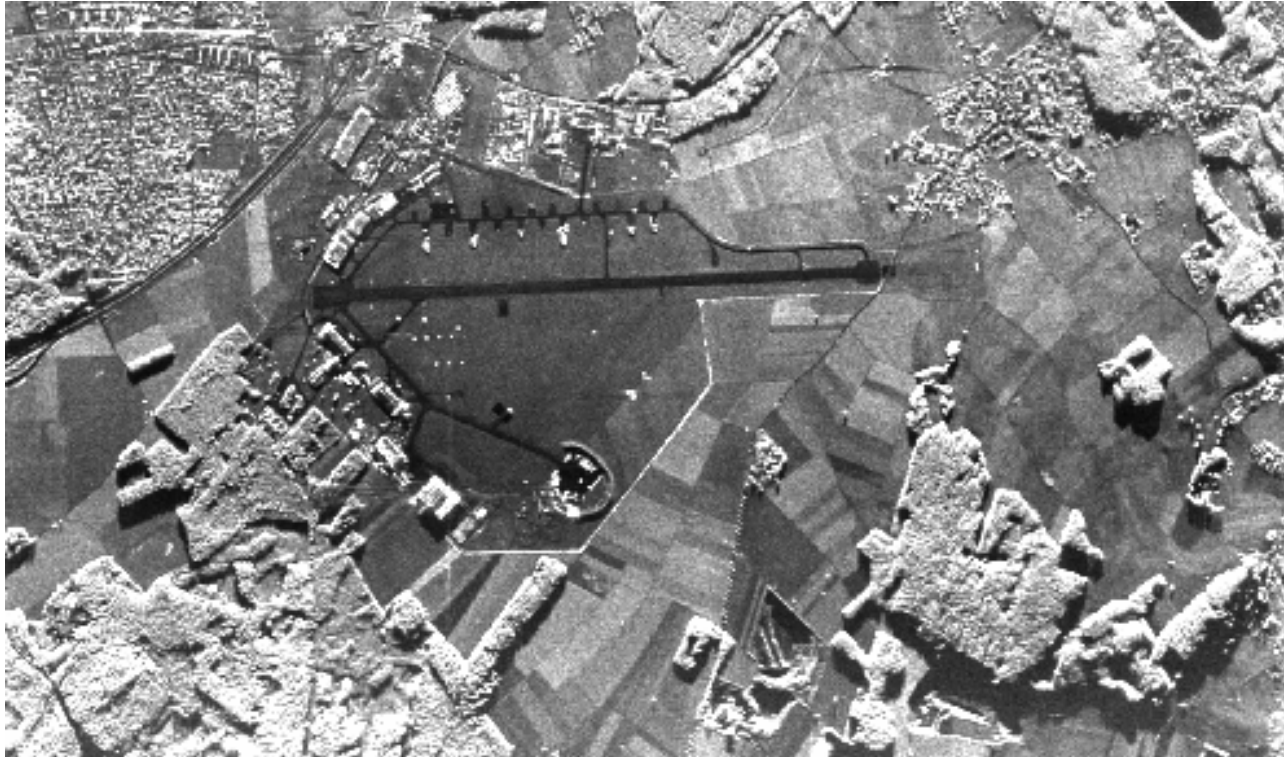
<http://www.neuro-fuzzy.de/nefclass/nefclassj>

Download NEFCLASS-J

Download the free version of NEFCLASS-J at <http://fuzzy.cs.uni-magdeburg.de>



Example: Line Filtering



- Extraction of edge segments (Burns' operator)
- Production net:
edges \rightarrow lines \rightarrow long lines \rightarrow parallel lines \rightarrow runways

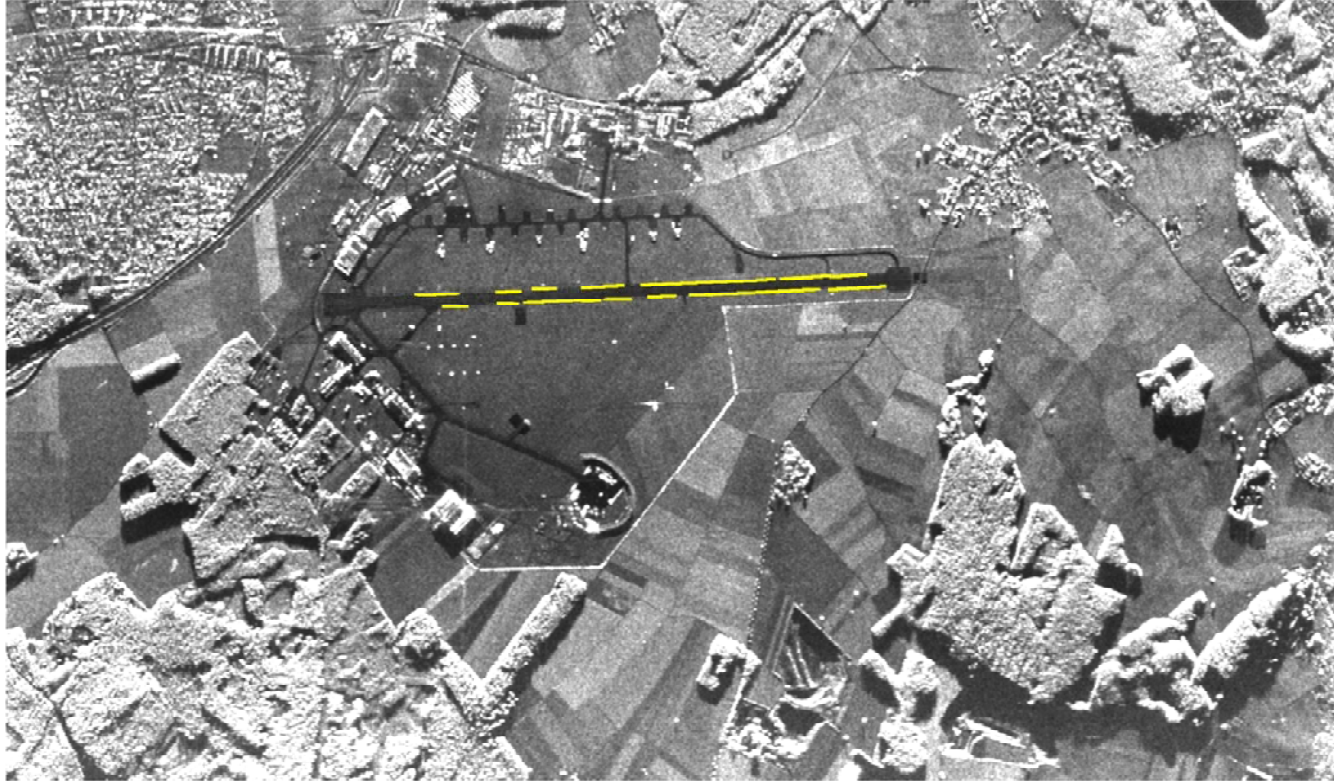
Example: Line Filtering



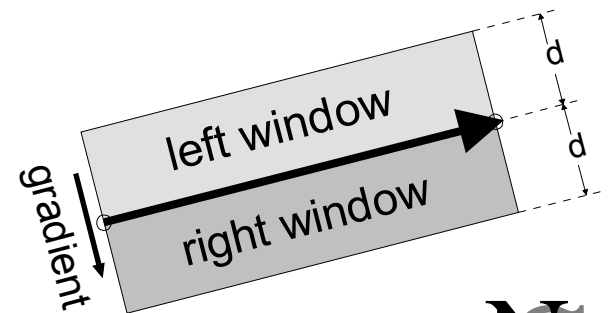
■ Problems

- extremely many lines due to distorted images
- long execution times of production net

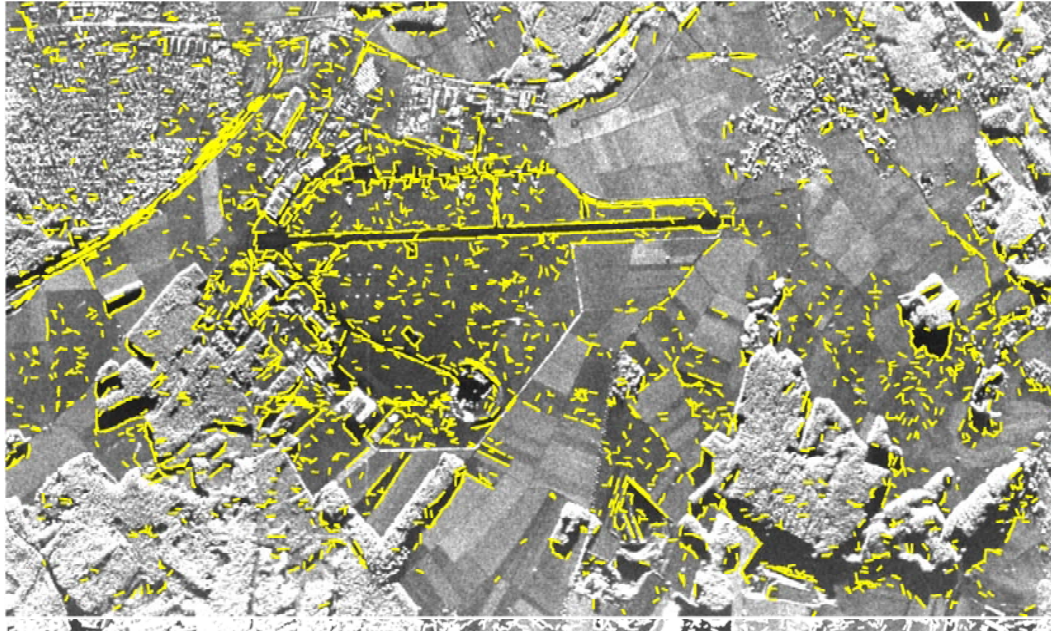
Example: Line Filtering



- Only few lines used for runway assembly
- Approach:
 - Extract textural features of lines
 - Identify and discard superfluous lines



Example: Line Filtering



- Several classifiers:
 - minimum distance, k-nearest neighbor, decision trees, NEFCLASS
- Problems: classes are overlapping and extremely unbalanced
- Result above with modified NEFCLASS:
 - all lines for runway construction found
 - reduction to 8.7% of edge segments

Example: Surface Quality Control

■ Today's Approach

The current surface quality control is done manually → an experienced worker treats the exterior surfaces with a grindstone. The experts classify surface form deviations by means of linguistic descriptions.

Cumbersome – Subjective - Error Prone Time Consuming

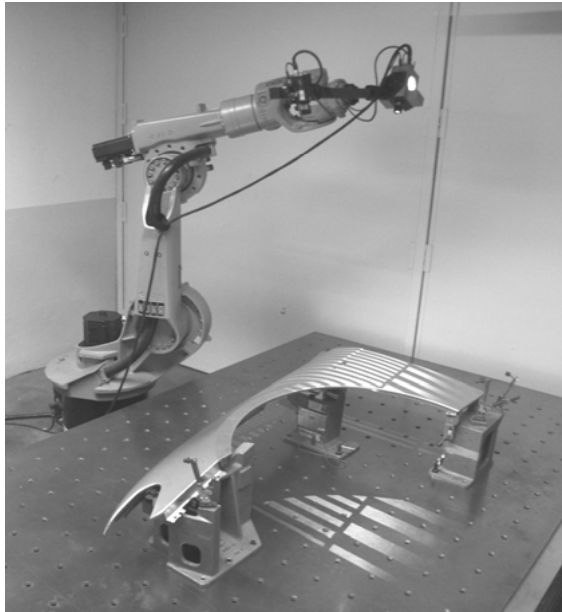


■ The Proposed Approach

Our Approach is based on the digitization of the exterior body panel surface with an optical measuring system.

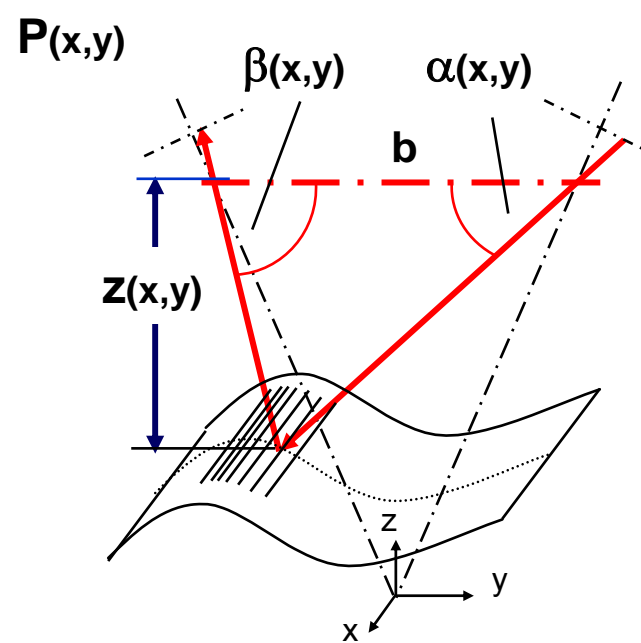
We characterize the form deviation by mathematical properties that are close to the subjective properties that the experts used in their linguistic description.

Topometric 3-D measuring system

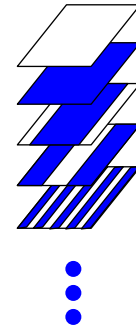


Triangulation and Gratings Projection

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Pixel coding



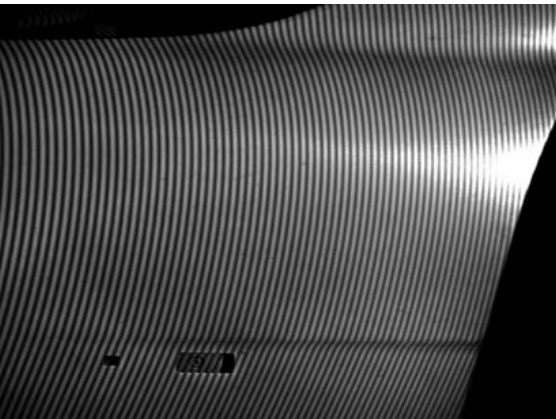
φ_n



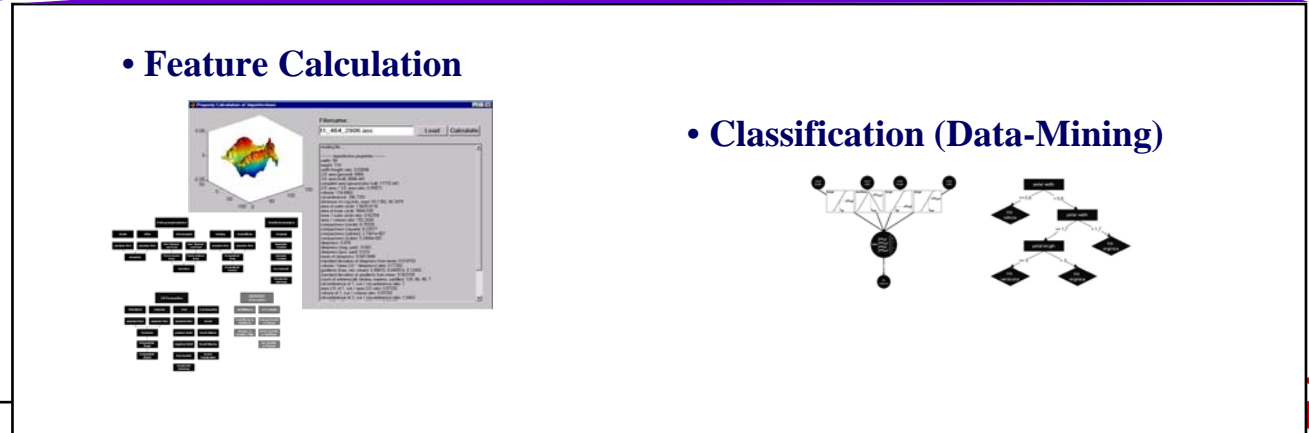
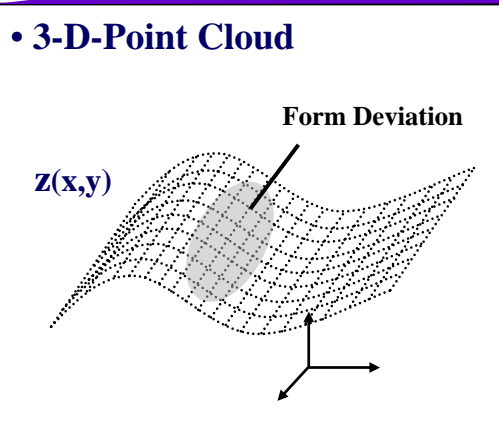
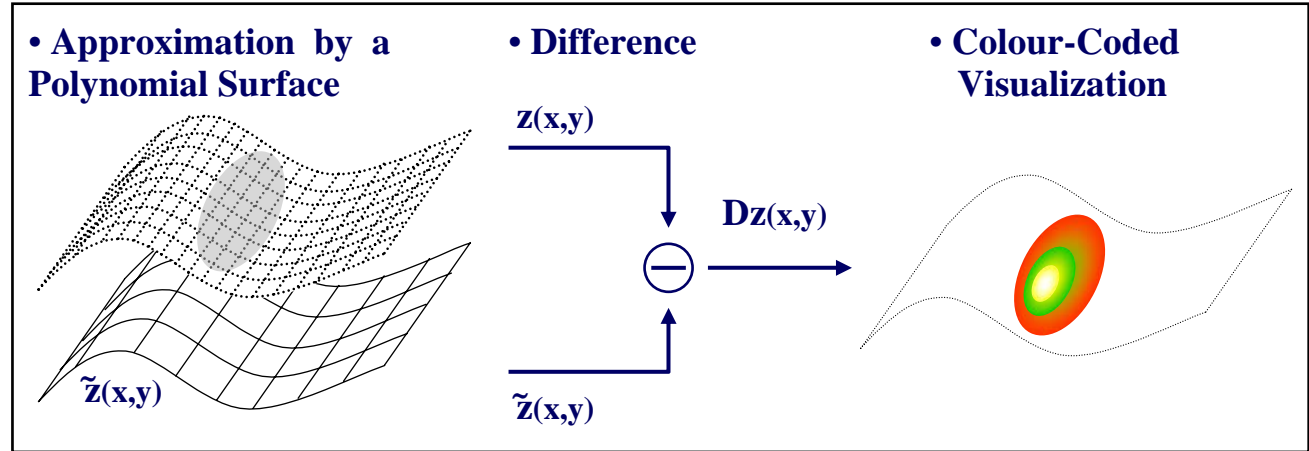
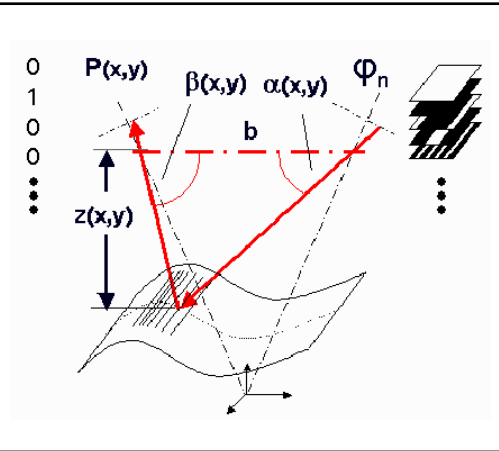
Miniaturized
Projection
Technique
(Grey Code
Phase shift)

- High Point Density
- Fast Data Collection
- Measurement Accuracy
- Contact less and Non-destructive

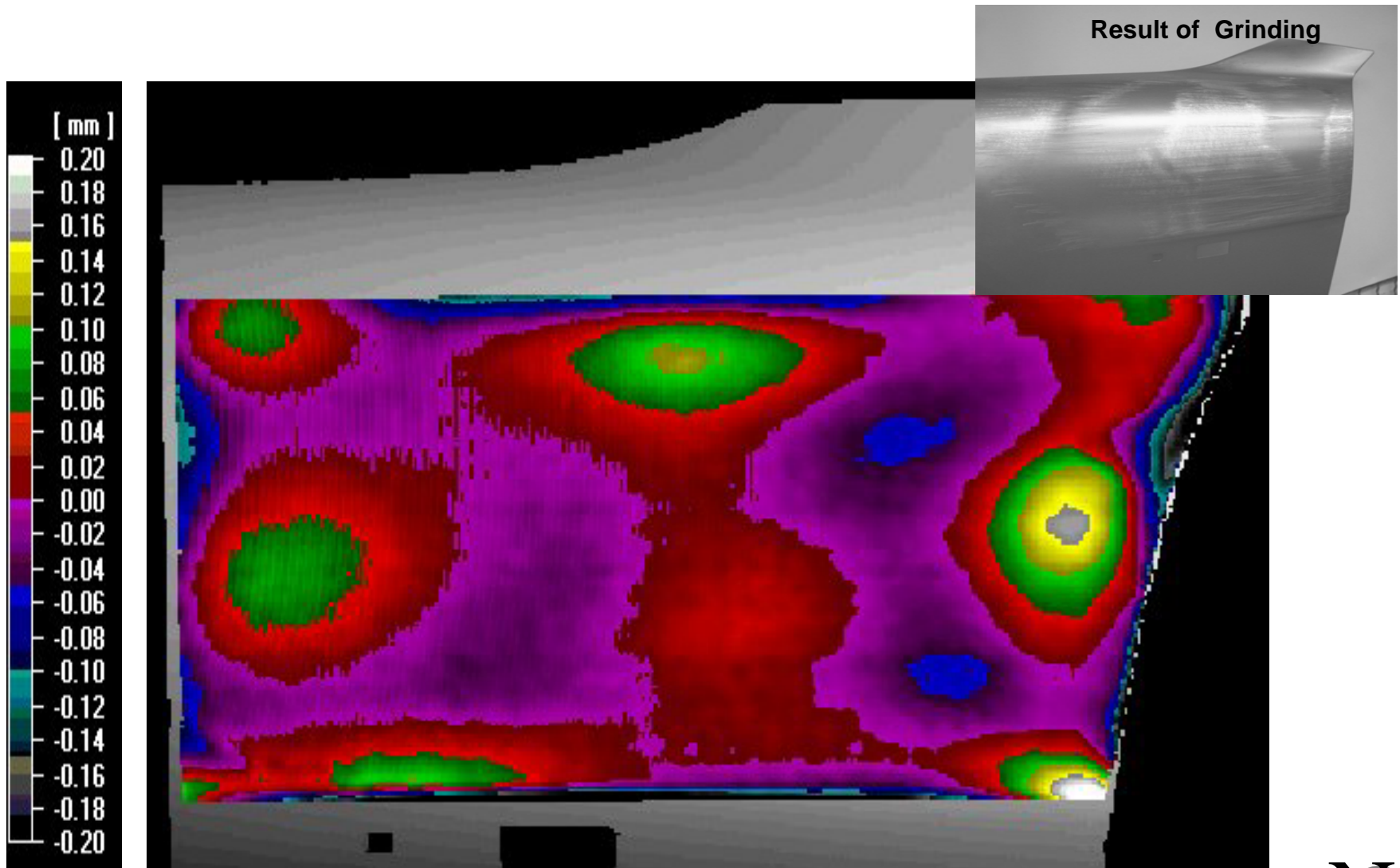
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Data Processing



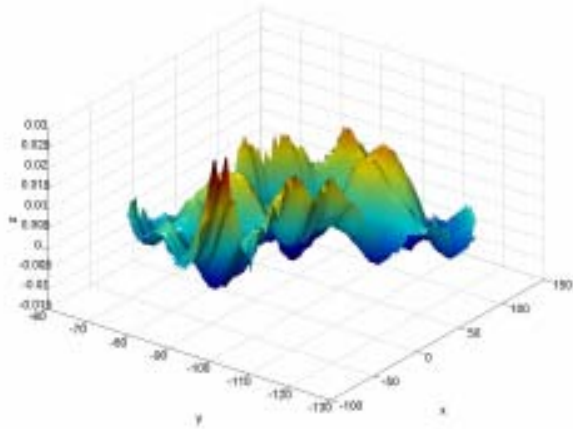
Color Coded Visualization



3D Visualization of Local Surface Defects

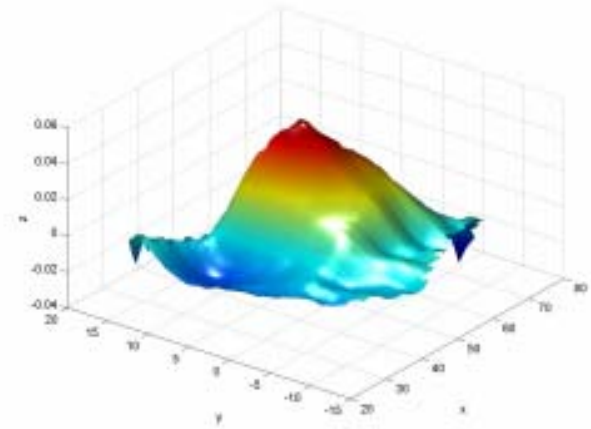
Uneven Surface

(several sink marks in series or adjoined)



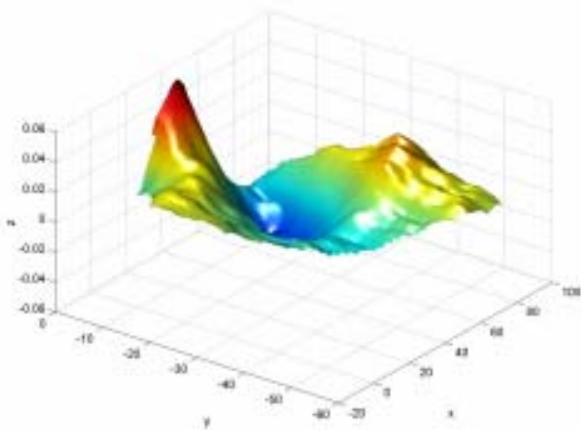
Press Mark

(local smoothing of (micro-)surface)



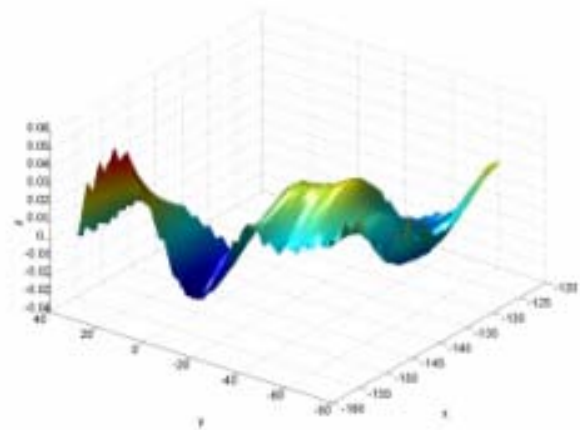
Sink Mark

(slight flat based depression inward)



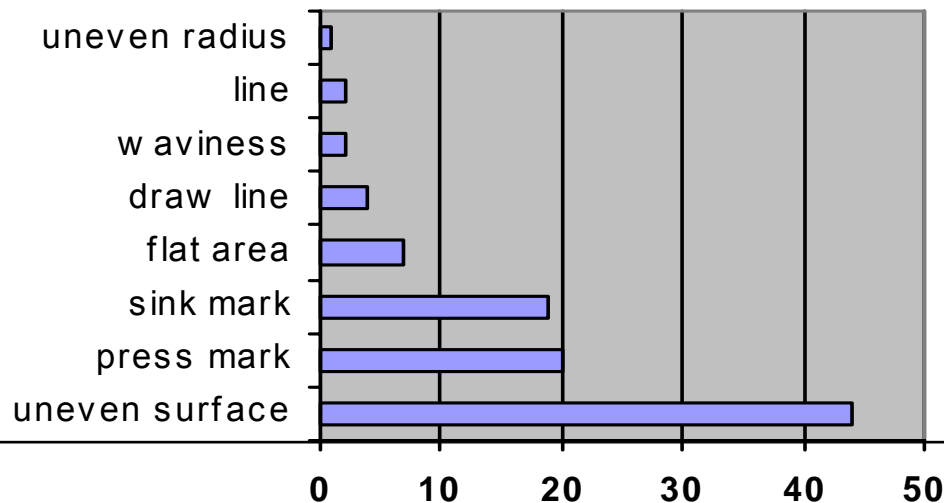
Waviness

(several heavier wrinklins in series)



Data Characteristics

- We analysed 9 master pieces with a total number of 99 defects
- For each defect we calculated 42 features
- The types are rather unbalanced
- We discarded the rare classes
- We discarded some of the extremely correlated features (31 features left)
- We ranked the 31 features by importance
- We use stratified 4-fold cross validation for the experiment.



Application and Results

The Rule Base for NEFCLASS

Rule base

- Rule 1: IF (max_distance_to_cog IS fun 2 AND min_extrema IS fun 1 AND max_extrema IS fun 1) THEN type IS press_mark
- Rule 2: IF (max_distance_to_cog IS fun 2 AND all_extrema IS fun 1 AND max_extrema IS fun 2) THEN type IS sink_mark
- Rule 3: IF (max_distance_to_cog IS fun 3 AND min_extrema IS fun 2 AND max_extrema IS fun 2) THEN type IS uneven_surface
- Rule 4: IF (max_distance_to_cog IS fun 2 AND min_extrema IS fun 2 AND max_extrema IS fun 2) THEN type IS uneven_surface
- Rule 5: IF (max_distance_to_cog IS fun 2 AND all_extrema IS fun 1 AND min_extrema IS fun 2) THEN type IS press_mark
- Rule 6: IF (max_distance_to_cog IS fun 3 AND all_extrema IS fun 2 AND max_extrema IS fun 3) THEN type IS uneven_surface
- Rule 7: IF (max_distance_to_cog IS fun 3 AND min_extrema IS fun 3) THEN type IS uneven_surface

Classification Accuracy

	NBC	DTree	NN	NEFCLASS	DC
Train Set	89.0%	94.7%	90%	81.6%	46.8%
Test Set	75.6%	75.6%	85.5%	79.9%	46.8%

Conclusions

- Neuro-Fuzzy-Systems can be useful for knowledge discovery.
- Interpretability enables plausibility checks and improves acceptance.
- (Neuro-)Fuzzy systems exploit tolerance for sub-optimal solutions.
- Neuro-fuzzy learning algorithms must observe constraints in order not to jeopardise the semantics of the model.
- Not an automatic model creator, the user must **work** with the tool.
- Simple learning techniques support explorative data analysis.